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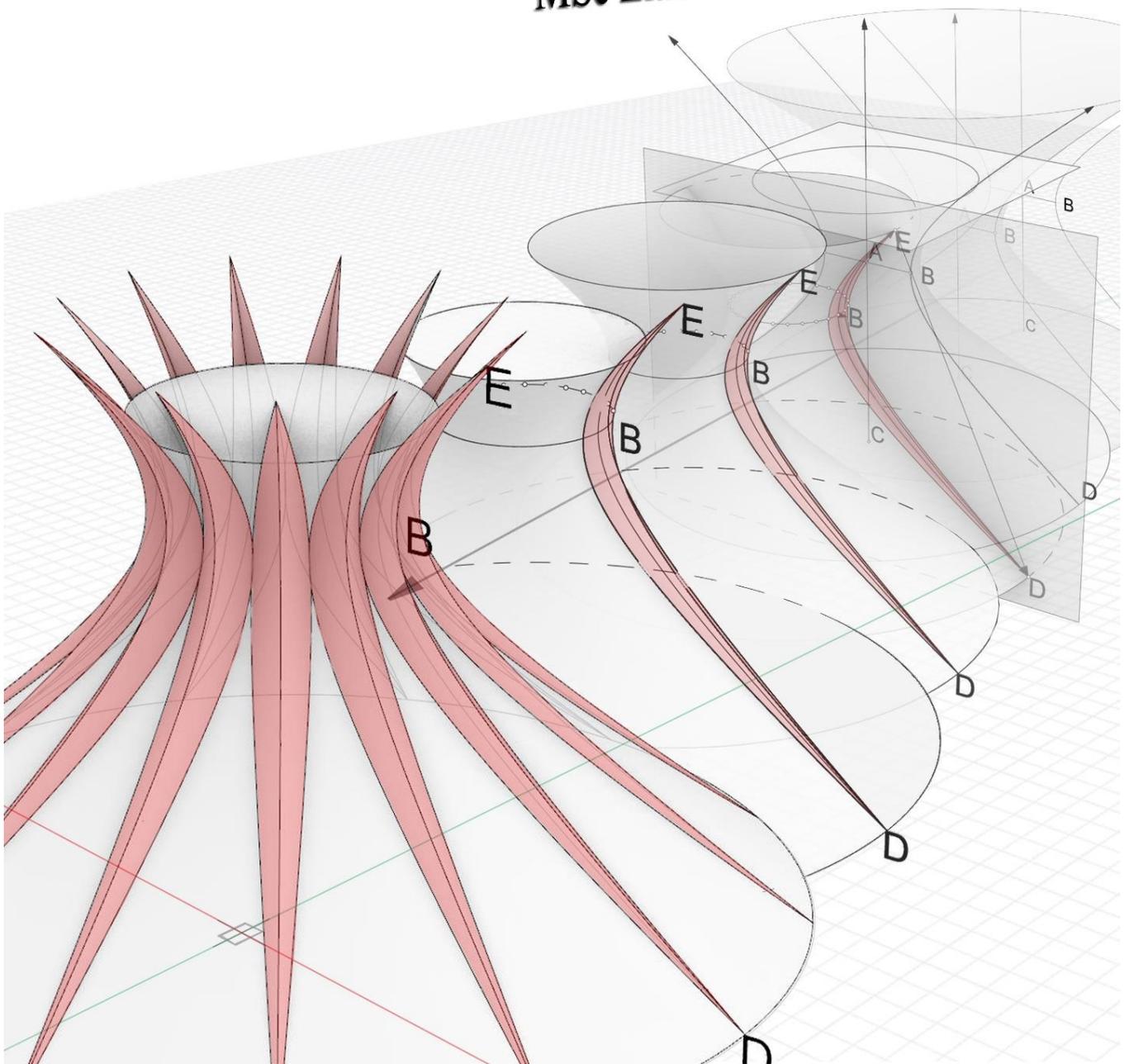
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# Geometrical surfaces in architecture – practice book

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# **GEOMETRIC SURFACES IN ARCHITECTURE – PRACTICE BOOK**

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## FOREWORD

Numerous architectural objects are proper works of art in the design sense; they not only represent their authors, but are recognizable features of cities and areas where they are located. Apart from design and aesthetic value, every architectural object has a use value, unlike other forms of art. In contemporary architectural design, freeform surfaces are more frequently applied for the objects designed for public use, because they are attractive and energy efficient. Moreover, parametric design is experiencing an expansion. Various geometric shapes can be used for architectural objects, but not all of them are suitable for each function. In order to design objects of complicated geometry, the architects must first learn the basic characteristics of geometric surfaces, and then their modeling, using one of contemporary CAD softwares.

That is why the Faculty of Civil Engineering and Architecture at the University of Niš, Serbia (further: FCEA) introduced an academic course Geometric Surfaces in Architecture (further: GSA) into the teaching plan of the master studies of Architecture in the academic year 2009/10. In theoretical classes, the students learned about all types and characteristics of geometric surfaces applied in architecture, while in practical classes they modelled those same surfaces in one of CAD softwares, although the modelling of especially curved geometric surfaces was very complicated and did not yield desired results. It was the students' opinion that the GSA course was very helpful and attractive to future architects, but that it should be taught earlier, so that the knowledge gained could be applied throughout their entire studies.

The authors of this practice book considered that the students should be introduced to geometric surfaces and their modelling in CAD software Rhinoceros at the very beginning of their studies, so that they could apply the gained knowledge in designing architectural objects in further studies, as well as later in practice. This software gives extraordinary opportunities for modelling both geometric and freeform surfaces.

Since the academic year 2016/17, the GSA academic course that the students frequent at the second year of their studies at the Architecture study program at FCEA Niš, includes theoretical as well as practical classes with 1+1 class ratio. In the theoretical part, the students are introduced to the classification of surfaces, ways of generating geometric surfaces, their application in architecture through history, using a large number of examples of derived objects. For the theoretical part of the course, there are presentations, as well as a monograph called *Geometric surfaces in Architecture*, published in 2012 by FCEA in Serbian language. Within the practical part of the course, the students are supposed to model architectural objects using geometric surfaces in Rhinoceros software. For the practice classes, the students were given templates that contain both textual explanations and drawings, which make object modelling easier for the students. The authors of this practice book were driven by the fact that the students do not have enough literature for the practice classes of the course, which is why the publishing of a practice book is of great importance.

Since one of the authors, Sonja Krasić, PhD, was a visiting professor at the Hosei University, Faculty of Engineering and Design in Tokyo, Japan, where she conducted the GSA course for the Japanese students of the second year of the Architecture study course, the need for the publishing of the practice book in English language emerged. The presentations for the theoretical part of the course were translated into English. By having this practice book published in English, architecture students worldwide will be able to improve their knowledge on geometric surfaces and their practical application, with a useful instruction on how to model them in CAD software Rhinoceros.

Due to a small number of teaching hours practice classes (45 minutes), in order for the students to learn how to model a larger number of geometric surfaces, which they are taught about during the theoretical lectures, one surface from the same lecture is processed during the practice classes, and another one for homework. While preparing the templates for practical exercises, the authors were driven by the idea that the exercises for practical work should be attractive architectural objects that can be modelled with as few options (commands) as possible, but also in different manners, so that the students would learn the software's possibilities better. With the textual explanations of the options, using the adopted geometrical terms, the drawings of important steps were also given, most frequently in perspective view,

with basic dimensions. For each assignment, whether done in class or for homework, the final shape of the object was shown in perspective and orthogonal projections, with all necessary dimensions. Didactical approach of step-by-step problem solving was applied during the making of all templates for practical work. Apart from the fact that the templates for practical learning help students learn object modeling more easily, with the use of remaining literature, they can also learn about geometrical surfaces that have found their application in architecture and have been used in making of the templates. The aim of the GSA course is for the acquired knowledge to be used in making of the final architectural projects in the GSA course itself, as well as in other design courses at the FCEA Nis.

*Geometric surfaces in architecture – practice book* will unify templates for practical learning, excerpts from theoretical classes and best examples of final students' projects at the GSA course at the FCEA Niš. Digital edition of the practice book in English language will be of great use to students and architects worldwide, in order for them to prepare for practice in contemporary circumstances.

The authors would like to thank the reviewers, Professor Naomi Ando and Professor Daniel Lordick, for the useful advice which contributed to the quality of the practice book being on the highest level. They would also like to thank MSc Milica Veljkovic who participated in creating the exercises templates. The authors would like to thank University library „Nikola Tesla” in Niš, who were understanding enough to publish this practice book as their digital edition.

Authors

# INTRODUCTION

Surfaces, like curves, are divided into empirical and regular. When the position of each point of the surface is determined by a certain law (rule), then the surface is regular, and they are divided into **analytic** and **geometric** (Table 1).

Table 1. Main division of surfaces

|               |           |           |             |
|---------------|-----------|-----------|-------------|
| SURFACES      | EMPIRICAL | FREEFORM  |             |
|               | REGULAR   | ANALYTIC  | ALGEBRAIC   |
|               |           |           | TRANSCEDENT |
|               |           | GEOMETRIC | POLYHEDRAL  |
|               |           |           | EXTRUSION   |
|               |           |           | HELICAL     |
|               |           |           | RULED       |
|               |           |           | ROTATIONAL  |
| TRANSLATIONAL |           |           |             |

Geometric surfaces which are generated by smooth motion of one line upon another are called **kinematic**. If a line "g" (straight or curve) moves according to some rule upon the second line "d" (straight or curve), then it forms a geometric surface.

Kinematic surfaces according to the type of movement **generatrix** "g" per the **directrix** (guideline) "d" are divided into:

**Translational surface** – generated by translation (parallel) movement of a curve g along the curve (straight line) d.

**Rotational surface** – generated by rotation of a curve (straight line) g around an axis o.

**Helical surface** – generated by rotation of some curve (straight line) g, around an axis o, with permanent translating along it.

When generatrix and directrix are straight lines (fig. 1), the resulting surface is developable, polyhedral. If the generatrix is a curve, and directrix is a straight line (fig. 2), the resulting surface is extrusion, single curved. When generatrix is a straight line and directrix is a curve (fig. 3), the resulting surface is a ruled, developable. If both generatrix and directrix are curves (fig. 4), the resulting surface is translational.

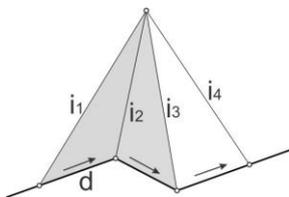


Figure 1

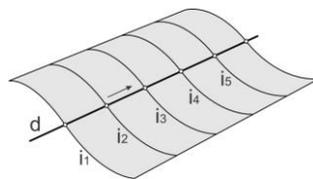


Figure 2

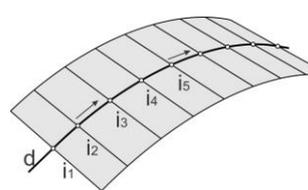


Figure 3

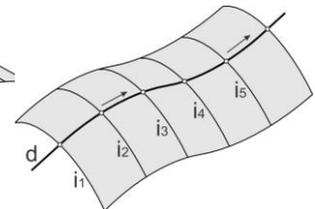


Figure 4

In addition to regular type, there is a special type of surface, empirical, that is generated by a series of matching or similar curves and they are called freeform surfaces.

Part of the space which is limited by one, two or more closed surfaces is a **solid**.

According to the classification of surfaces which are used in technical practice, especially in architecture, they are divided into two basic groups (Table 1): geometric (regular) and free-form (empirical) surfaces.

Traditional classes of geometric surfaces based on Helmut Pottmann in Architectural geometry text book are shown in Table 2:

**Table 2.** Basic division of geometric surfaces

| GEOMETRIC SURFACES (Traditional surface classes - H. Pottmann) |   |
|--|---|
| I POLYHEDRAL   | 1. REGULAR POLYHEDRONS<br>2. POLYHEDRONS  |
| II EXTRUSION   | 1. POLYHEDRONS<br>2. SINGLE CURVED SURFACES   |
| III HELICAL  | 1. HELICAL SURFACES with curve generatrix<br>2. HELICOIDS with line generatrix                    |
| IV RULED   | 1. DEVELOPABLE<br>2. NON-DEVELOPABLE<br>2.1. One generatrix system<br>2.2. Two generatrix systems |
| V ROTATIONAL   | 1. GENERAL SURFACES (QUADRICS):<br>2. SPECIAL SURFACES<br>3. GENERAL TIPE OF ROTATIONAL SURFACE   |
| VI TRANSLATIONAL   | 1. GENERAL<br>2. SPECIAL  |

Table 3 gives all classes of geometric surfaces. Some of the surfaces belong to two, or three classes at the same time, because they can be generated using two different types of movement.

**Table 3.** Division of geometric surfaces

| I POLYHEDRAL SURFACES                         | V ROTATIONAL SURFACES                       |
|---|---|
| I 1. REGULAR POLYHEDRONS                      | V 1. GENERAL ROTATIONAL SURFACES (QUADRICS) |
| I 1.1. Platonic solids                        | V 1.1. Rotational ellipsoids                |
| I 1.2. Archimedean solids                     | V 1.2. Rotational paraboloids               |
| <b>I.2. POLYHEDRONS</b>                       | V 1.3. Two-sheet rotational hyperboloids    |
| I 2.1. Pyramids                               | V 1.4. One-sheet rotational hyperboloids    |
| I 2.2. Prisms                                 | V 2. SPECIAL ROTATIONAL SURFACES            |
| <b>II EXTRUSION SURFACES</b>                  | V 2.1. Spheres                              |
| II 1. Cones, Pyramids (central extrusion)     | V 2.2. Cones                                |
| II 2. Cylinders, Prisms (parallel extrusion)  | V 2.3. Cylinders                            |
| <b>III HELICAL SURFACES</b>                   | V 2.4. Torus                                |
| III 1. Helical surfaces with curve generatrix | V 3. GENERAL TIPE OF ROTATIONAL SURFACE     |
| III 2. Helicoids with line generatrix         | <b>VI TRANSLATIONAL SURFACES</b>            |
| <b>IV RULED SURFACES</b>                      | VI 1. GENERAL TRANSLATIONAL SURFACES        |
| <b>IV 1. DEVELOPABLE</b>                      | VI 2. SPECIAL TRANSLATIONAL SURFACES        |
| IV 1.1. Pyramids, Prisms                      | VI 2.1. Elliptic paraboloids                |
| IV 1.2. Cones, Cylinders                      | VI 2.2. Hyperbolic paraboloids              |
| IV 1.3. Tangent surfaces of a space curve     | VI 2.3. Two-sheet elliptic hyperboloids     |
| <b>IV 2. NON DEVELOPABLE:</b>                 |   |
| IV 2.1. ONE GENERATRIX SYSTEM                 |   |
| IV 2.1.1. Conoids                             |   |
| IV 2.1.2. Cylindroids                         |   |
| IV 2.1.3. Tetroids                            |   |
| IV 2.2. TWO GENERATRIX SYSTEM                 |   |
| IV 2.2.1. One-sheet rotational hyperboloids   |   |
| IV 2.2.2. Hyperbolic paraboloids              |   |

All geometric surfaces classes have found vast application in architecture, especially in 20<sup>th</sup> and 21<sup>st</sup> century. The students are introduced to the characteristics and methods of generating all geometric surfaces, as well as the best-known examples of architectural objects on which they were applied, during the theoretical classes of the academic course Geometric Surfaces in Architecture (GSA). Out of large number of architectural objects on which geometric surfaces were applied, one object per surface was chosen (the most attractive one, in the opinion of the authors of this practice book), for students to model in the Rhinoceros software, during the practical classes of the GSA course.

## CONTENTS OF THE GSA ACADEMIC COURSE

GSA academic course at FCEA Niš is performed in 12 units at the theoretical and 12 units at practical classes. The contents of the course include all types of geometric surfaces, and the units were thusly conceived that one unit encompasses the same kind of surfaces.

In theoretical classes, the first unit is introductory, with clearly marked aim of the course; the second one gives the classification of surfaces and the historical overview, and from the third unit onwards the students learn about the geometric surfaces according to the division given in 2<sup>nd</sup> unit (Table 4), with characteristics and examples from architectural practice, both finished objects and the best students' projects from this course since its beginning.

**Table 4.** Contents of lectures in course GSA on FCEA in Nis (Source: Krasic et al. 2018)

|        |   |         |  |
|--------|---|---------|--|
| Unit 1 | Introduction to the course                                    | Unit 7  | Helical surfaces with curve and line generatrix: Helicoids   |
| Unit 2 | Classification of the surfaces                                | Unit 8  | Ruled non-developable surfaces with one generatrix system: Cylindroids, Conoids and Tetroids                             |
| Unit 3 | Regular polyhedrons: Platonic and Archimedean solids          | Unit 9  | Ruled non-developable surfaces with two generatrix systems: One-sheet rotational hyperboloids and Hyperbolic paraboloids |
| Unit 4 | Polyhedral surfaces: Pyramids and Prisms                      | Unit 10 | Special rotational surfaces: Spheres   |
| Unit 5 | Extrusion surfaces: Cones and Cylinders                       | Unit 11 | General rotational surfaces: Ellipsoids, Paraboloids and Two-sheet hyperboloids  |
| Unit 6 | Ruled developable surfaces: Tangent surfaces of a space curve | Unit 12 | General type of rotational surfaces, special rotational surface-Torus and Translational surfaces                         |

The practical classes foresee 12 units (Table 5). First, it is necessary for students to get introduced to the functioning of the computer software Rhinoceros, since this course is their first contact with it. In the first three units, basic commands used for modelling geometric surfaces are explained, and those are the commands for drawing and transformation of geometric elements, as well as various settings for the drawing process. From the 4<sup>th</sup> unit the students model the surfaces according to the division given in the Unit 2 during the theoretical classes (Table 1). One surface is modelled in the laboratory, and the second surface of the same type for homework, so that the students can be introduced to ways of modelling as many surfaces as possible.

**Table 5.** Contents of practice classes in course GSA on FCEA in Nis (Source: Krasic et al. 2018)

|        |   |         |  |
|--------|---|---------|--|
| Unit 1 | Rhinoceros – Introduction, points, lines and polygons                                   | Unit 7  | Rhinoceros – Helical surfaces with line generatrix: Oblique helical staircase and normal helical staircase (homework)  |
| Unit 2 | Rhinoceros – Free-form curves, circles, ellipses, parabolas, hyperboles, arcs and helix | Unit 8  | Rhinoceros – Ruled non developable surfaces with one generatrix system: Conoidal canopy and cylindroid (homework)  |
| Unit 3 | Rhinoceros – Surfaces and geometric solids  | Unit 9  | Rhinoceros – Ruled non developable surfaces with two generatrix systems: Hyperbolic paraboloid's object and special rotational surfaces spherical canopy (homework)        |
| Unit 4 | Rhinoceros – Platonic solids: Pentagonal dodecahedron and icosahedron (homework)        | Unit 10 | Rhinoceros – Rectilinear ruled surfaces with two generatrix systems: One sheet rotational hyperboloid's object and special rotational surfaces: spherical slice (homework) |
| Unit 5 | Rhinoceros – Polyhedral surfaces: Pyramidal folds and prismatic folds (homework)        | Unit 11 | Rhinoceros – General rotational surfaces: Triaxial ellipsoid's object and rotational paraboloid's object (homework)  |
| Unit 6 | Rhinoceros – Extrusion surfaces: Conical folds and cylindrical object (homework)        | Unit 12 | Rhinoceros – Special rotational surface: Torus's object and Translational surface (homework)   |

For the practical classes, templates were made, in which modelling of the objects was shown both in text and in drawing, step by step, with clearly marked views in which certain steps should be performed, including the data whether a command should be chosen from the

main or mini menu, or from the command line. The templates text also includes the data whether the option Osnap should be turned on, which is used for precise positioning of points and setting tangents and perpendiculars to curves (End, Point, Mid, Cen, Int, Perp, Tan...), as well as the options Grid Snap, Ortho, Planar, Smart Track, which are turned on as needed, and which serve for easier manipulation while drawing geometric elements. While creating the tasks for modelling geometric surfaces, the most optimal options were taken into account, but at the same time, for each new task, a new modelling option was chosen when more possible modelling options were available. In this way, the students are better acquainted with software possibilities.

## **FINAL PROJECT ON THE GSA COURSE**

Based on theoretical and practical work, as the final project, each student should model an original architectural object (or three elements of either indoor or outdoor furniture), from one type of geometric surface, as directed by the teacher. Architectural object (or furniture piece) should be put in appropriate surroundings and visualized. By placing the object in the adequate environment and applying the materials, its realistic representation is obtained. The final architectural project is handed in on a poster, size 50x70cm, which should contain perspective views of the object, orthogonal projections and characteristic steps using which the final object was obtained (drawings most often in perspective view from the Rhinoceros software), especially if planar intersections or mutual surfaces intersections were used for the object.

## **CONTENTS OF THE PRACTICE BOOK**

The practice book contains theoretical excerpts, exercise templates and homework, information about working in the Rhinoceros software as well as examples of best students' final projects at the GSA course, for each unit foreseen during the practical classes.

# UNIT 1 – RHINOCEROS: INTRODUCTION, POINTS, LINES AND POLYGONS

## EXERCISE 1

EXERCISE 1 explains the position and use of the basic commands (options) in the software Rhinoceros for: starting the Rhinoceros software; using the work space with 4 windows (**Top, Front, Right and Perspective view**, fig. 5); giving commands on the command line (or via main menu or mini menu, which are easier to visually follow options for drawing, fig. 6 and 7); diminishing (enlarging) the drawing; movement of the drawing in the window; setting measurement units in the program; using the wireframe and orthogonal direction for drawing; precise drawing; returning to the previous or next step during the drawing (**undo, redo**); classification of certain parts of the drawing in layers depending on colour, type of the line, texture and thickness (fig. 8); handling during the drawing process (repeating or interrupting the command, selection of one or more objects, deselection).

Moreover, Unit 1 explains which options in main or auxiliary (mini) menu are used to draw one or more points (fig. 9), lines, polylines (fig. 10 and 11), tangents and normal to curves. Then it is explained using which options from either menu it is possible to draw rectangle and regular polygons (triangles, quadrangles, pentagons, etc., fig. 12). Using the example of simple polygons, it is explained how they could be separated into single elements (**Explode**), and then merged into one whole (**Join**) and how the intersection of planar figures can be found (**Curve/From Objects/Intersection**).

It was also explained how to use the commands for transforming geometric elements in the main menu (lines, curves, polygons...fig.13), as well as in the mini menu (fig. 14), because it is the simpler way: movement (**Move**), copying (**Copy**), mirroring (**Mirror**), rotation (**Rotate**), splitting (**Split**), erasing the parts (**Trim**), extension of elements (**Extend**), enlargement/diminishing (**Scale**).

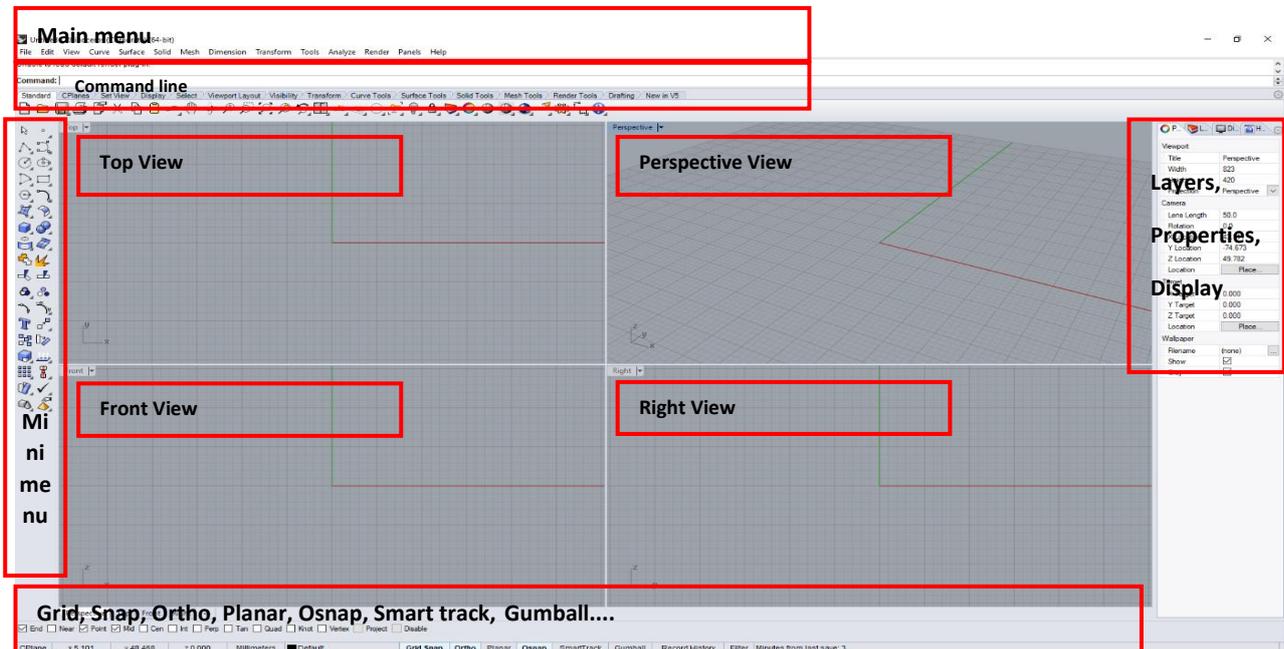


Figure 5 – Desktop software Rhinoceros



Figure 6 – Main menu commands

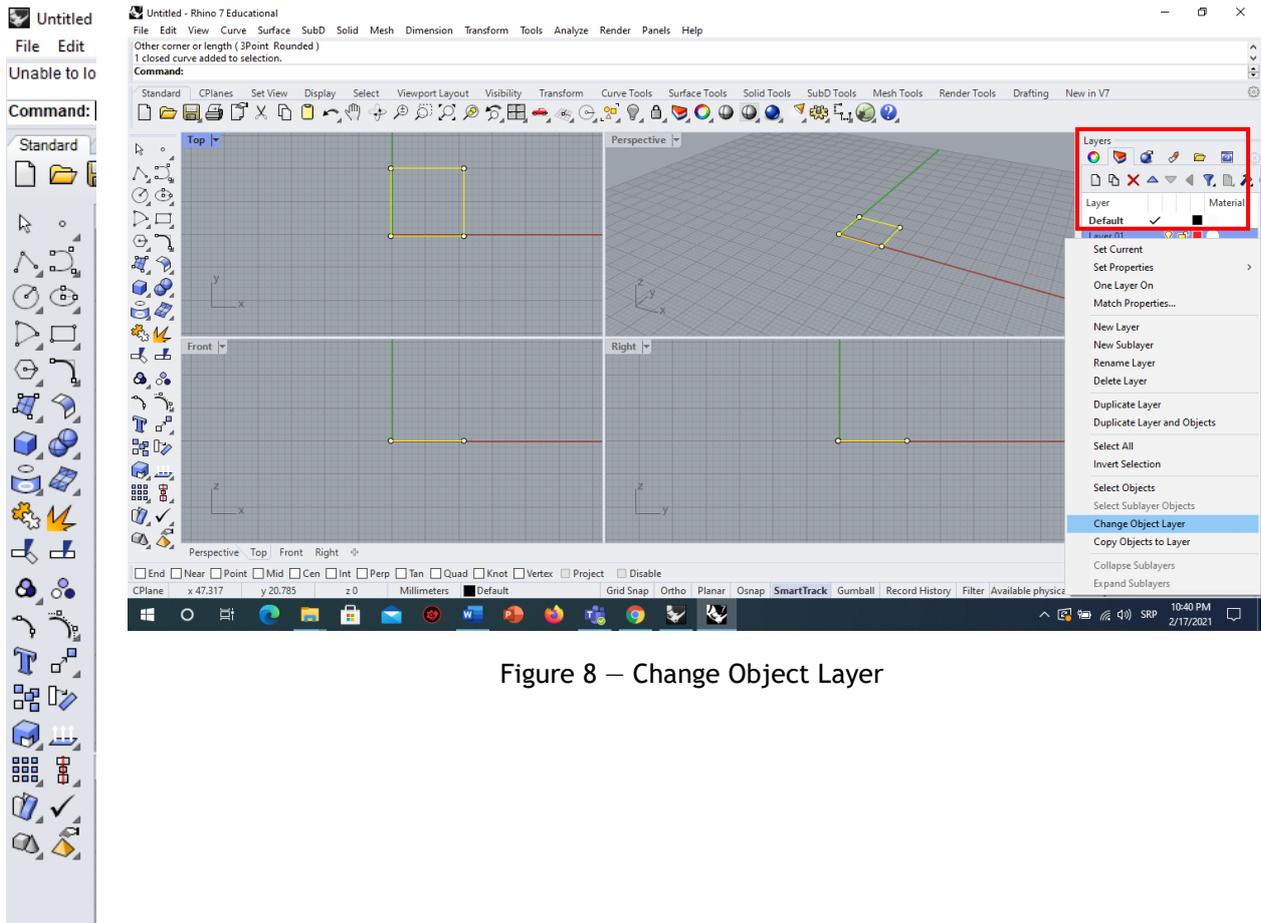


Figure 8 – Change Object Layer

Figure 7 – Mini menu commands

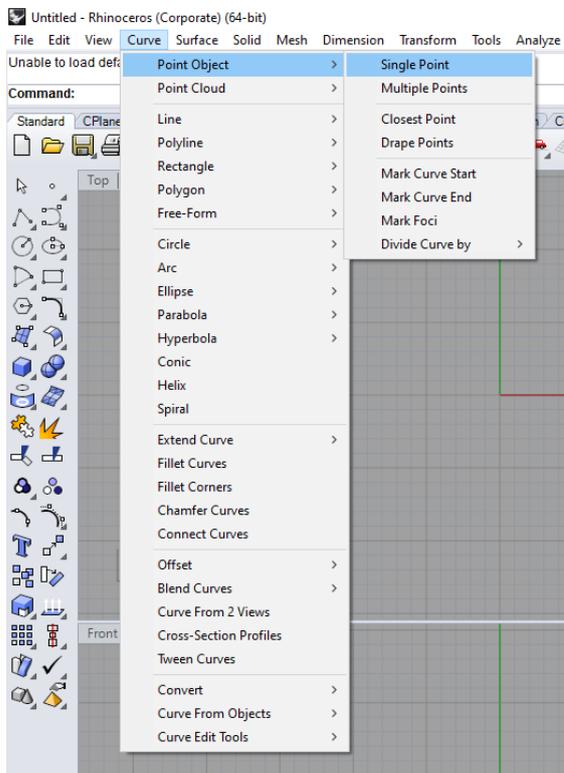


Figure 9 – Curve/Point object

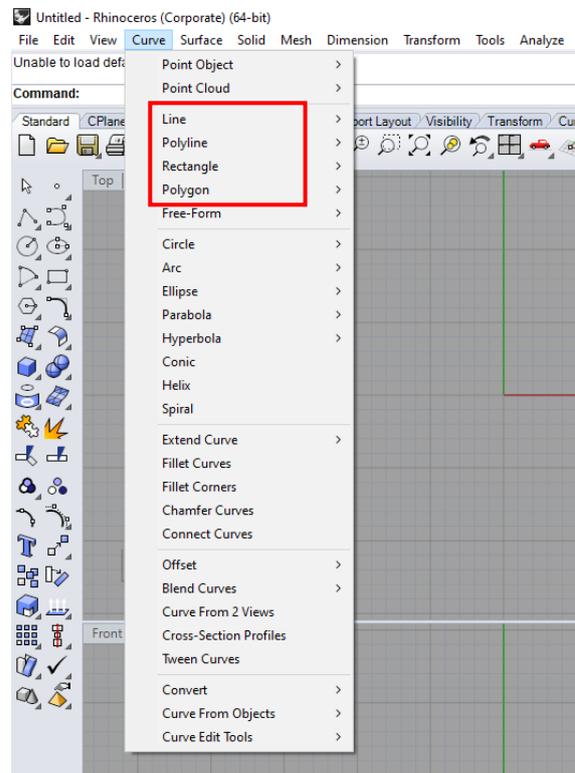


Figure 10 – Curve/Line, Polygon

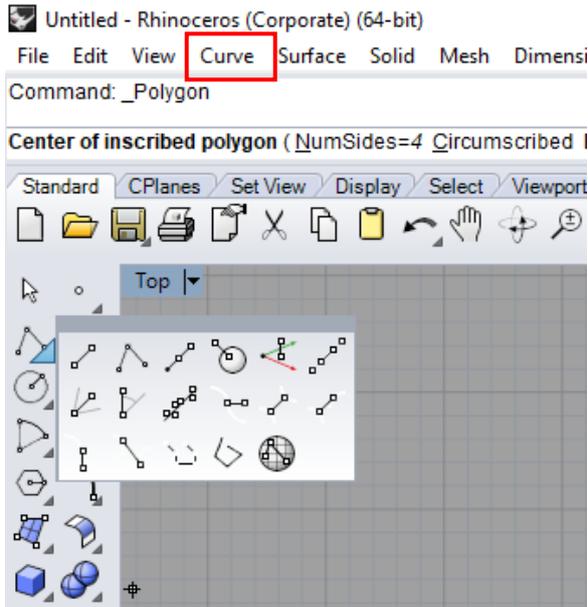


Figure 11 – Curve/Line

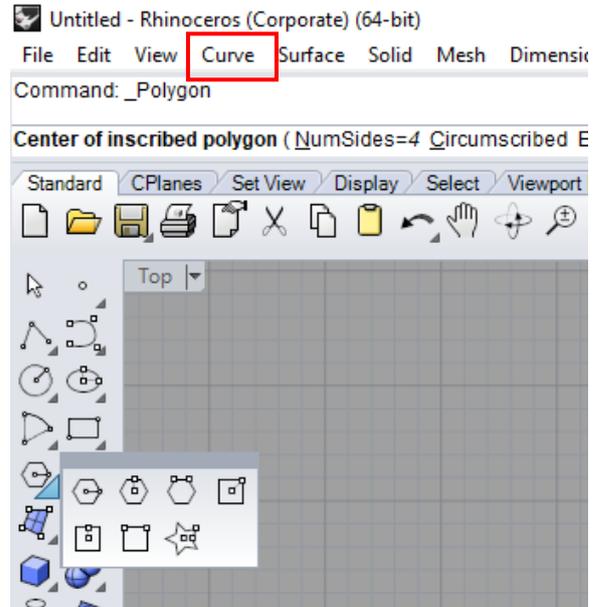


Figure 12 – Curve/Polygon

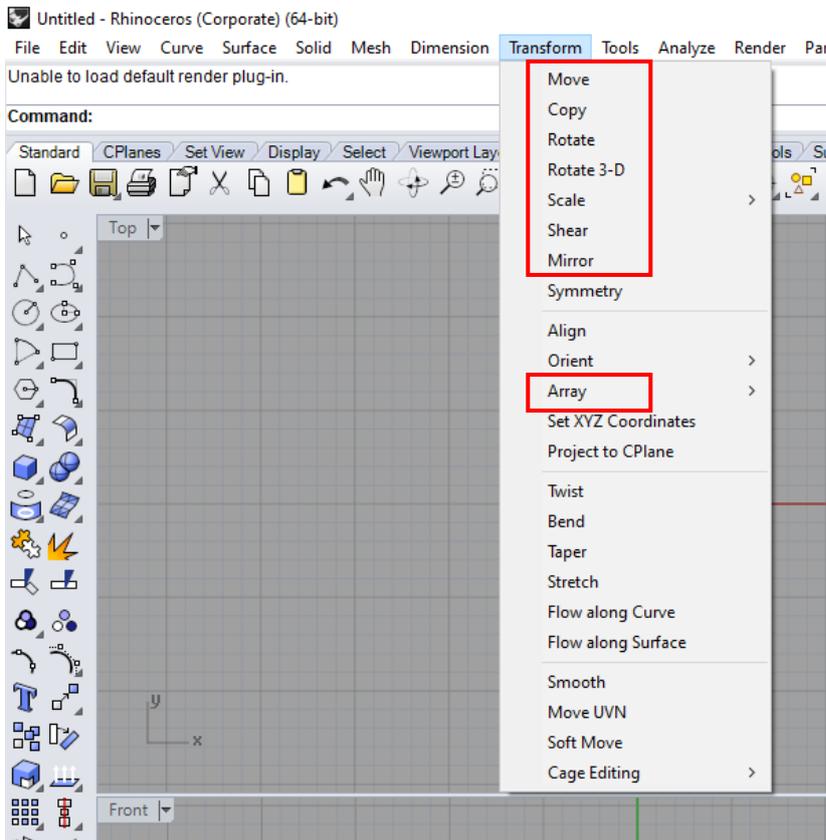


Figure 13 – Transform/Move/Copy/Rotate/Scale/Mirror

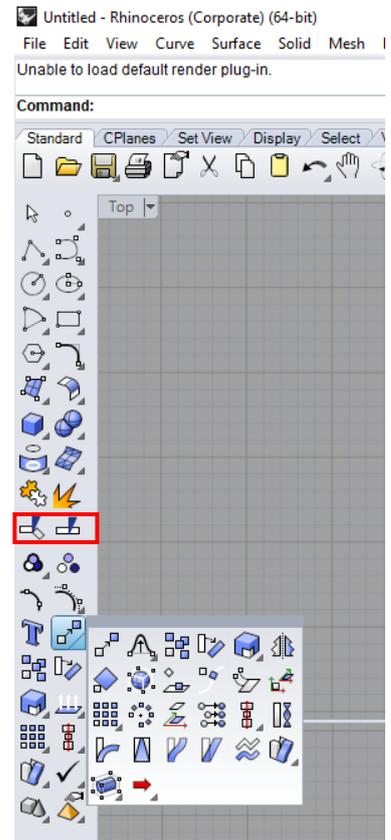


Figure 14 – Transform/Mini Menu

## HOMEWORK 1

HOMEWORK 1 is to draw 9 points, given with  $x$ ,  $y$  and  $z$  coordinates, taking care to keep the computer mouse positioned in TOP or PERSPECTIVE view. If the mouse is positioned in FRONT view, the coordinates are placed in order  $x$ ,  $z$ ,  $y$ .

## EXERCISE 1 – Rhinoceros: Drawing points, lines and polygons

### Starting the Rhinoceros (64-bit):

- Click on the icon Rhinoceros. Opens the menu and work space, which consists of 4 windows: **Top, Front, Right and Perspective**;
- Enlarging the window – double click on the window name;
- Choosing the window in which we want to work – left click.

### Important Commands:

- Enlarging (**zoom in**) and diminishing (**zoom out**) of the drawing – the scroll wheel of the mouse;
- Movement – right click/+Shift and move the cursor (simultaneously);
- Setting the unit for drawing: **File/Properties/Document Properties/Units**
- Command line – giving commands by typing;
- **Grid Snap** – capture points of the network are plotted in projection windows;
- **Ortho** – draw a line by parallel axes that are marked in red and green line;
- **Osnap** – precise drawing, capturing certain points on the drawing;
- **Undo and Redo**;

### Layers (Standard/Edit Layers) – classification of certain parts of the drawing depending on the color, line type, texture and thickness:

- **New layer** – add a new layer;
- **Sublayer** – sublevels of a layer;
- **Change Object Layer** – the subsequent changes in the layer of a plotted object (object selection/right click on a future layer/Change Object Layer);
- Temporary turning off of a layer is done by clicking on the light bulb next to layer (On);
- **Lock** – lock layer;
- **Current** – active Layer;

### Commands to handle when drawing in the program:

- Repeat the command – right click or the Space on the keyboard.
- Canceling commands – right click or Enter on the keyboard.
- Selection of one object – left click;
- Selection of multiple objects one by one – left click + Shift
- Deselection – left click + Ctrl;
- Selection from left to right – marking the objects in the whole framed by selection;
- Selection from right to left – mark all objects that are partially included;

### Point:

1. **Curve/Point Object/Single Point** – drawing one point;
2. **Mini Menu/Single Point** – drawing one point (other way);
3. **Curve/Point Object/Multiple Points** – drawing multipoint;
4. **Mini Menu/Multiple Points** – drawing multipoint (other way);

### Line:

1. **Curve/Line/Single Line** – drawing one line by assigning the length in the command line;

2. **Curve/Line/Line Segments** – drawing polygons consisting of individual lines;
3. **Curve/Line/Polyline** – drawing polygons as a whole;
4. **Mini Menu/Explode** – disassembling object on particular items;
5. **Curve/Line/Rectangle** – drawing quadrangle;
6. **Curve/Line/Polygon** – drawing a polygon;
7. **Curve/From Objects/Intersection** – curve in the intersection of two lines/surfaces/solids;

**Mini Menu:** (Tools/Options/Toolbars/File/Import Toolbar/default.rui/Open)

- Move
- Copy
- Mirror
- Rotate
- Split
- Join
- Trim
- Extend
- Scale

**HOMEWORK 1: Drawing points with assigning the exact coordinates\***

- Draw points (by using options **Curve/Text Object**) given by coordinates and examine their projections and position in space (in command line enter the coordinates separated by dots) **O(0.0. 0)**, **A(5.0.0)**, **B(-3.0.0)**, **C(0.4.0)**, **D(0.0.7)**, **E(2.3. 0)**, **F(4.0.7)**, **G(5.6.7)**, **H(3.-7.-5)**.

*\*Make sure that the mouse is positioned in Top or Perspective view. If the mouse is positioned in the Front view, then the coordinates are entered in order x,z,y.*

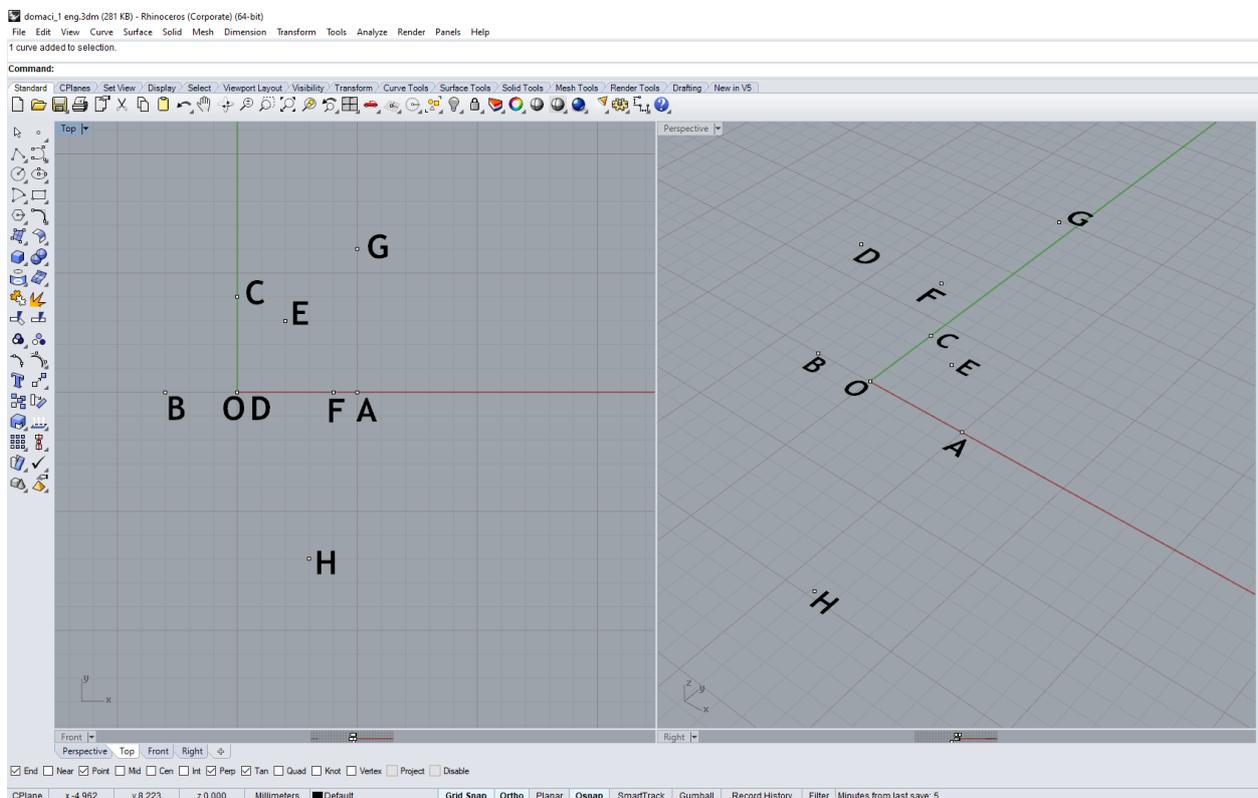


Figure 15 – Example Homework 1

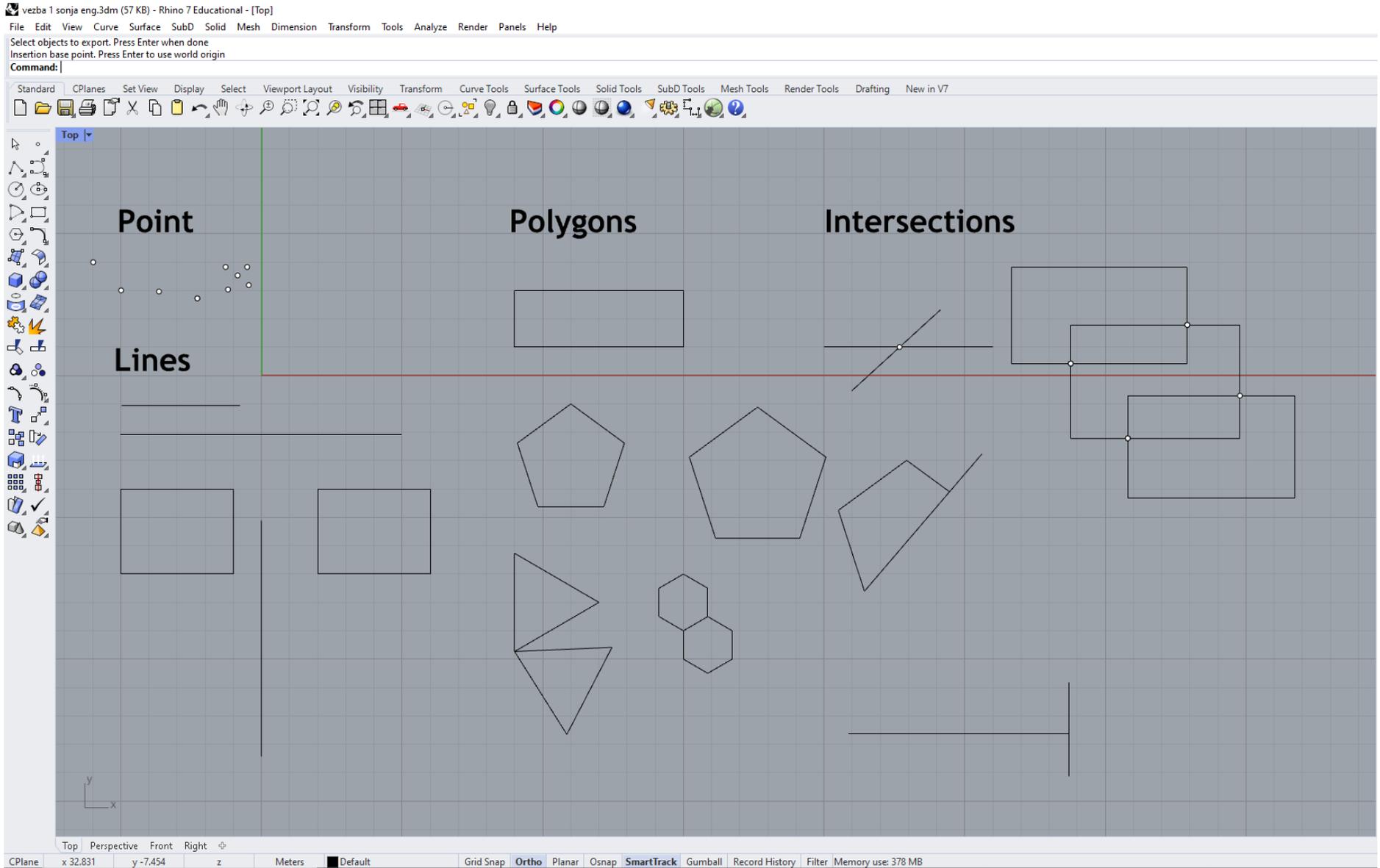


Figure 16 – Example Exercise 1

# UNIT 2 – RHINOCEROS: FREE-FORM CURVES, CIRCLES, ELLIPSES, PARABOLAS, HYPERBOLAS, ARCS AND HELICES

## EXERCISE 2

EXERCISE 2 explains how to construct planar free-form curves, conics of II degree (circles, ellipses, parabolas and hyperbolas) and arcs in the Rhinoceros software. It is shown how to construct helices and spirals from spatial curves. There are different constructions of planar free-form curves and conics of II degree. In this unit, only some constructions were chosen for explanation, because they are most frequently used in practice. For modeling planar curves (conics of II degree) commands from mini menu (fig. 17, 18 and 19), or from the main menu (fig. 20, 21 and 22) can be used. Only the constructions for parabola and hyperbola in the mini menu are found in the option for free-form curves (fig. 19). There are multiple ways of constructing arcs, only three most characteristic ones are shown (fig. 23). Spatial curves of helix and spiral have only one construction that is located in the main menu (fig. 24).

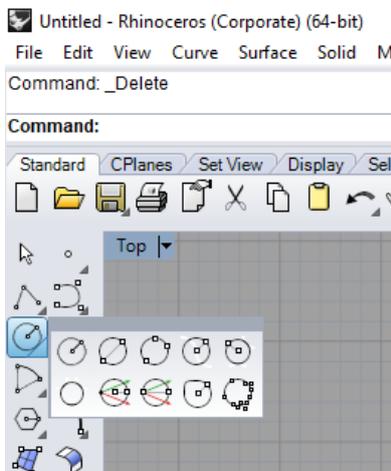


Figure 17 – Curve/Circle

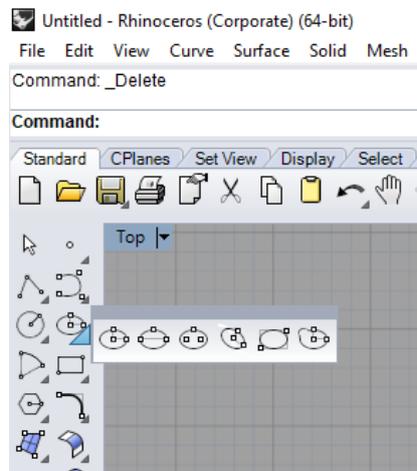


Figure 18 – Curve/Ellipse

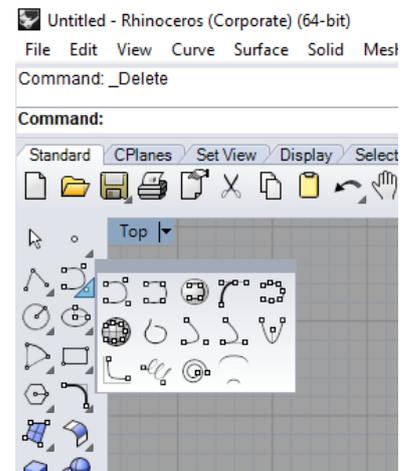


Figure 19 – Curve/Free-form curve

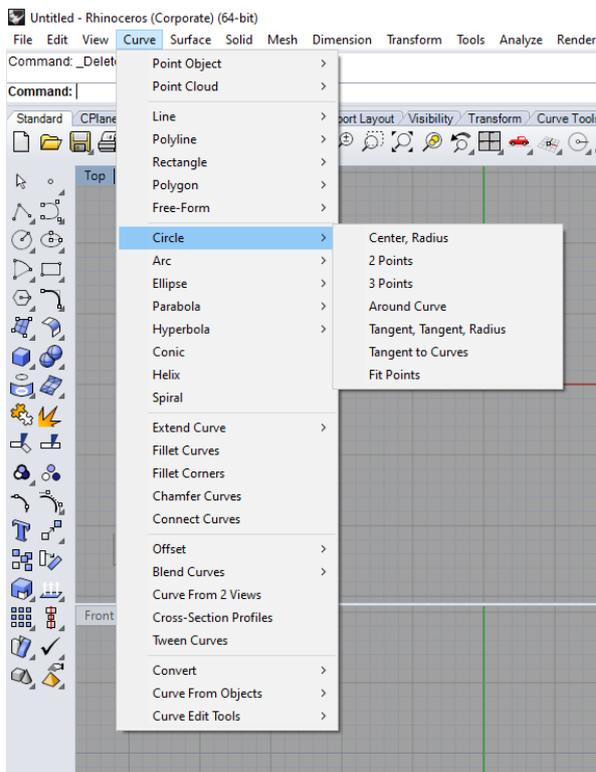


Figure 20 – Curve/Circle

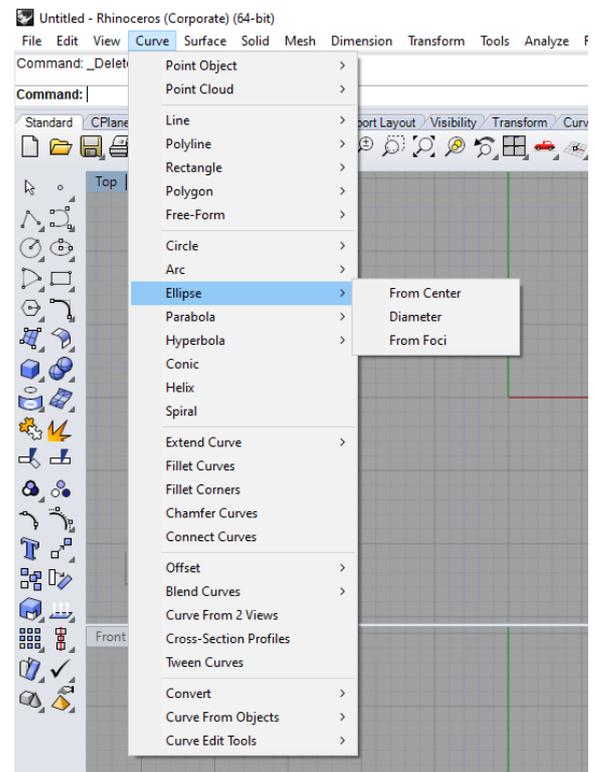


Figure 21 – Curve/Ellipse

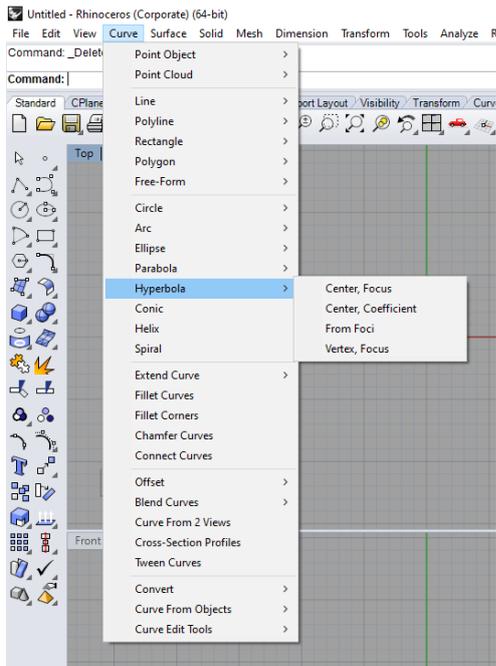


Figure 22 – Curve/Hyperbola

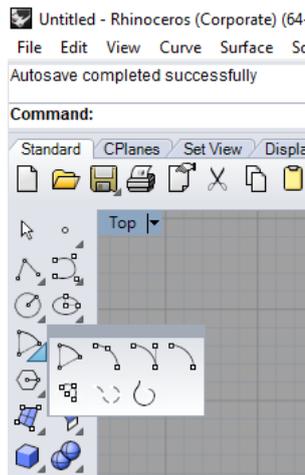


Figure 23 – Curve/Arc

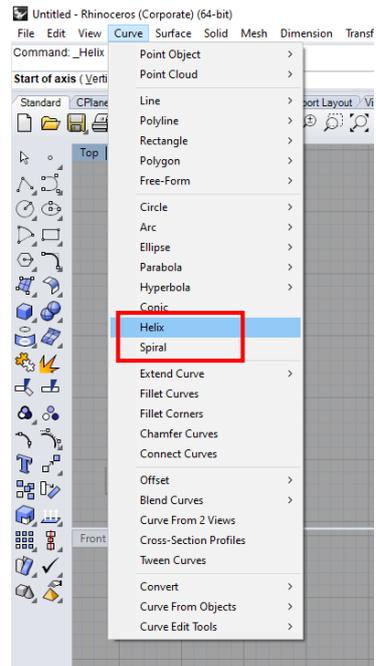


Figure 24 – Curve/Helix/Spiral

Then, different operations with curves are shown: division into equal lengths of the segments or equal number of parts (fig. 25); cutting, either mutually or with a surface or a solid, determining solid contour in a specific direction and from a given distance, adding the generatrix in a specific direction from an arbitrary distance, making of the wire model from a solid (fig. 26). This unit also shows how lines or open curves can be extended (fig. 27) as well as free-form curves (fig. 28).

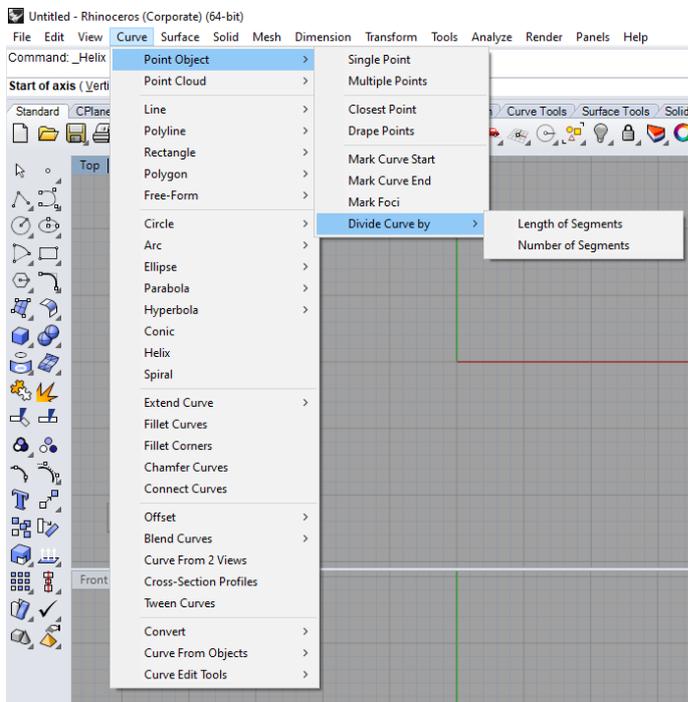


Figure 25 – Curve/Point object/Divide Curve by

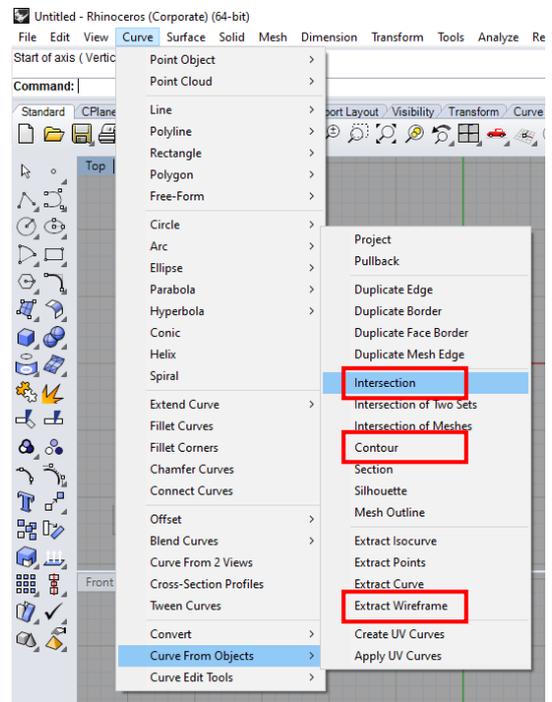


Figure 26 – Curve/Curve from object

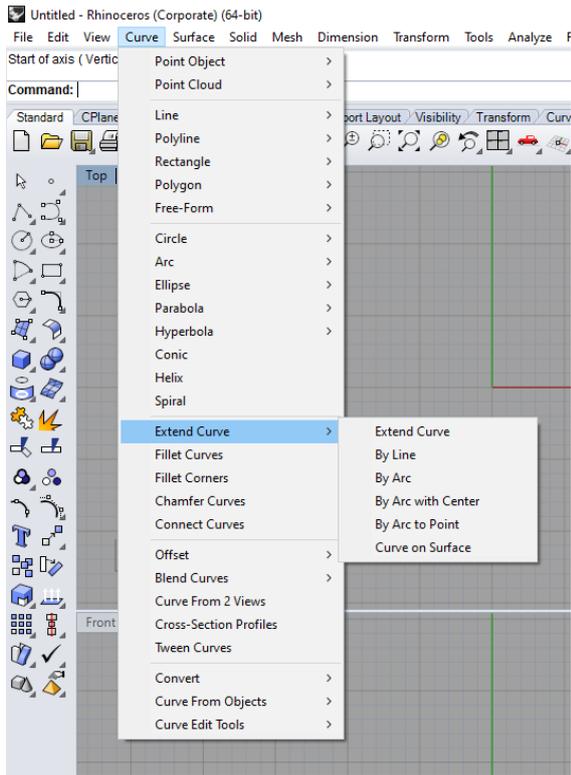


Figure 27 – Curve/Extend Curve

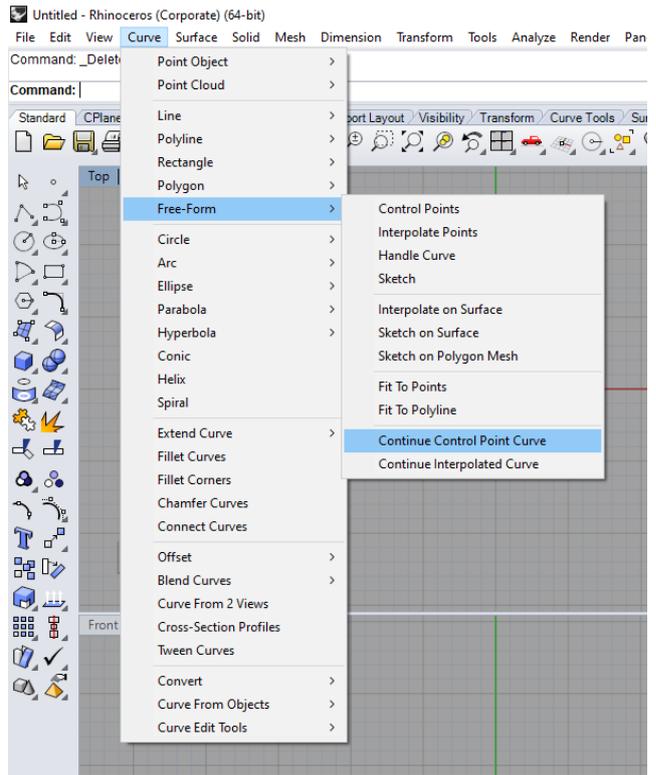


Figure 28 – Curve/Continue control point curve

## HOMEWORK 2

HOMEWORK 2 is to replace (in the given basis of the object) lines with curves, whose constructions are shown.

## EXERCISE 2 – Rhinoceros: Drawing free-form curves, circles, ellipses, parabolas, hyperbolas, arcs and helices

### Procedures for constructing curves (free-form curve, circle, ellipse, parabola, hyperbola, arc and helix)

1. **Curve/Free-Form/Interpolate Points** – free-form curve using curve points;
2. **Curve/Free-Form/Fit to Polyline** – free-form curve using lines (tangent in curve point)
3. **Curve/Circle/Center, Radius** – circle by using center and radius
4. **Curve/Circle/2 Points** – circle by using diameter;
5. **Curve/Circle/3 Points** – circle through three points on the circle;
6. **Curve/Circle/Around Curve** – circle perpendicular to the line or curve;
7. **Curve/Ellipse/From Center** – ellipse by using both axes and center;
8. **Curve/Ellipse/Diameter** – ellipse by using the length of one axis;
9. **Curve/Ellipse/From Foci** – ellipse by using foci
10. **Mini Menu/Ellipse/Around Curve** – ellipse perpendicular to the line;
11. **Mini Menu/Ellipse/By Corners** – ellipse by using tangent parallelogram
12. **Curve/Parabola/Focus, Direction** – parabola by using focus and axis;
13. **Curve/Parabola/Vertex, Focus** – parabola by using vertex and focus;
14. **Curve/Hyperbola/Center, Focus** – hyperbola by using center and focus;
15. **Curve/Hyperbola/From Foci** – hyperbola by using two foci
16. **Curve/Arc/Center, Start, Angle** – arc by using angle between two branches;
17. **Curve/Arc/Start, End, Point** – arc by using tendon and center;
18. **Curve/Arc/Start, Point, End** – arc by using points (each click denotes a single point of arc, there are three);
19. **Curve/Helix** – define axis and radius of cylinder basis;
20. **Curve/Spiral**;

### Processing the card "Curve":

- **Point object/Divide Curve by/Number/Segments**
- **Free-form/Continue control point curve;**
- **Extend Curve**

### Curve from object

1. **Intersection (of Two Sets/of Two Meshes)** – intersection of lines/curves/surfaces;
2. **Contour** – find solid contour in a certain direction at a given distance;
3. **Extract Isocurve** – add generatrix in a certain direction at an arbitrary distance;
4. **Extract Wireframe** – make a wired model of the solid;

## HOMEWORK 2: Drawing curves in given basis of the object

- On the given basis of the object (fig. 29), change internal or outer walls (arbitrarily), using the options from the exercises.

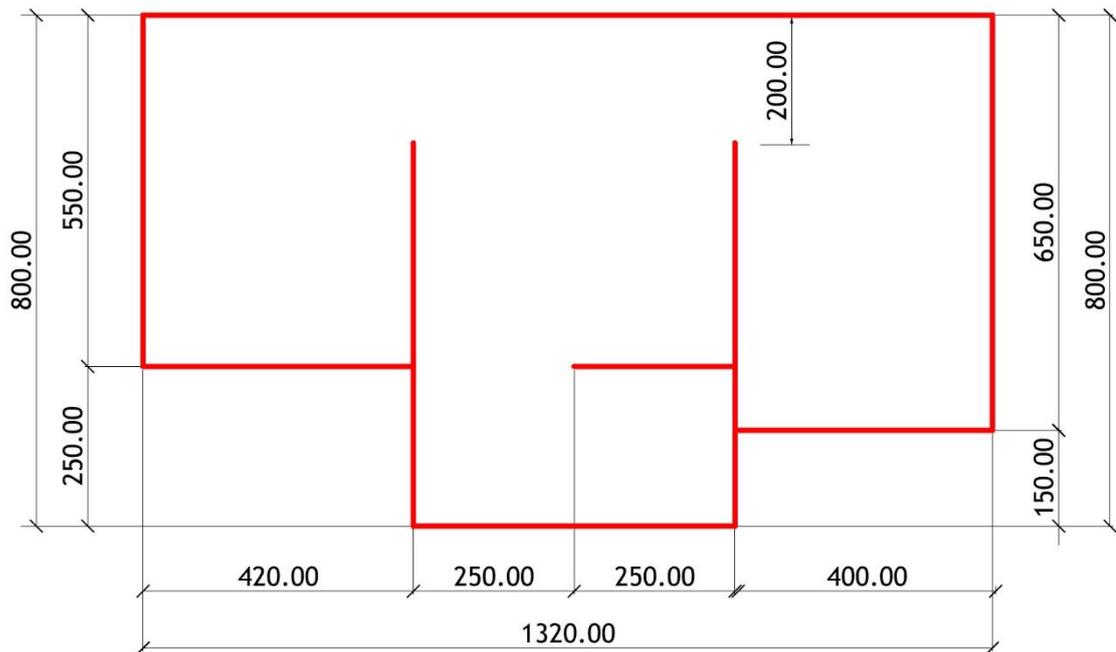


Figure 29 – Basis of the object

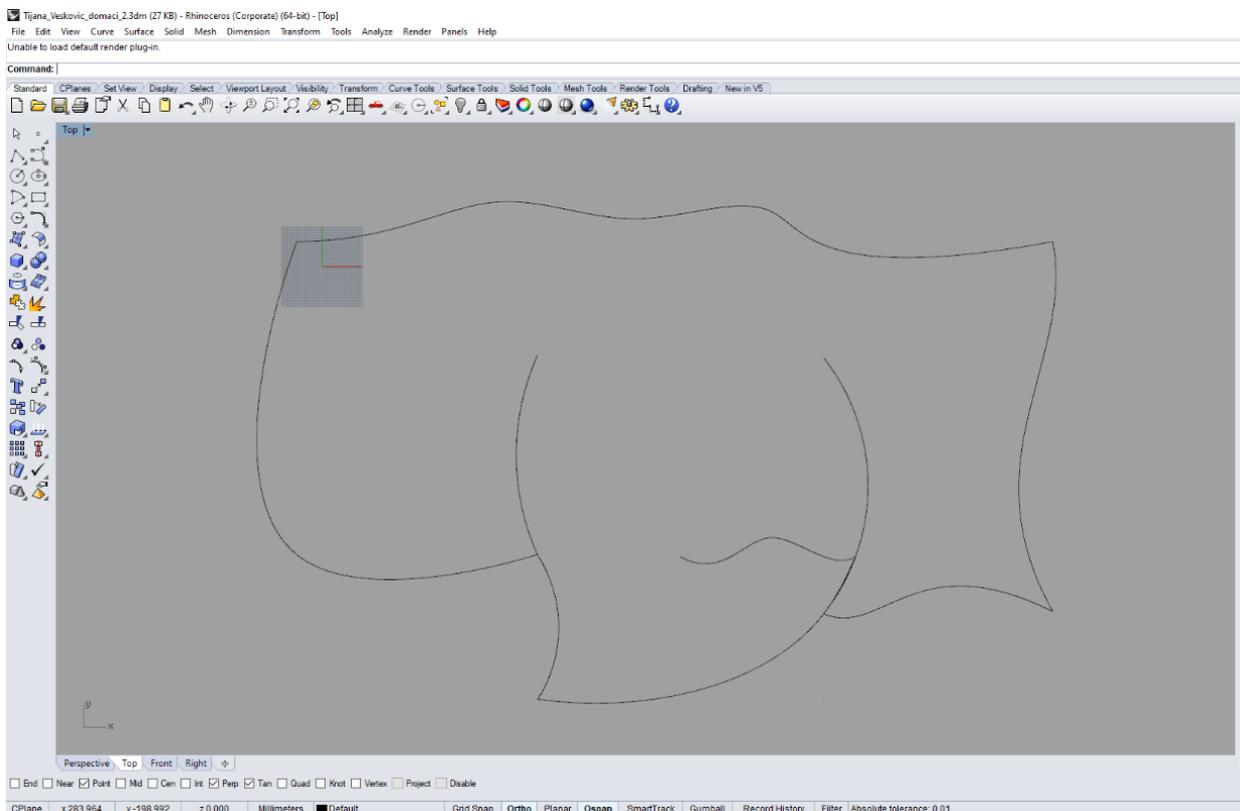


Figure 30 – Example Homework 2

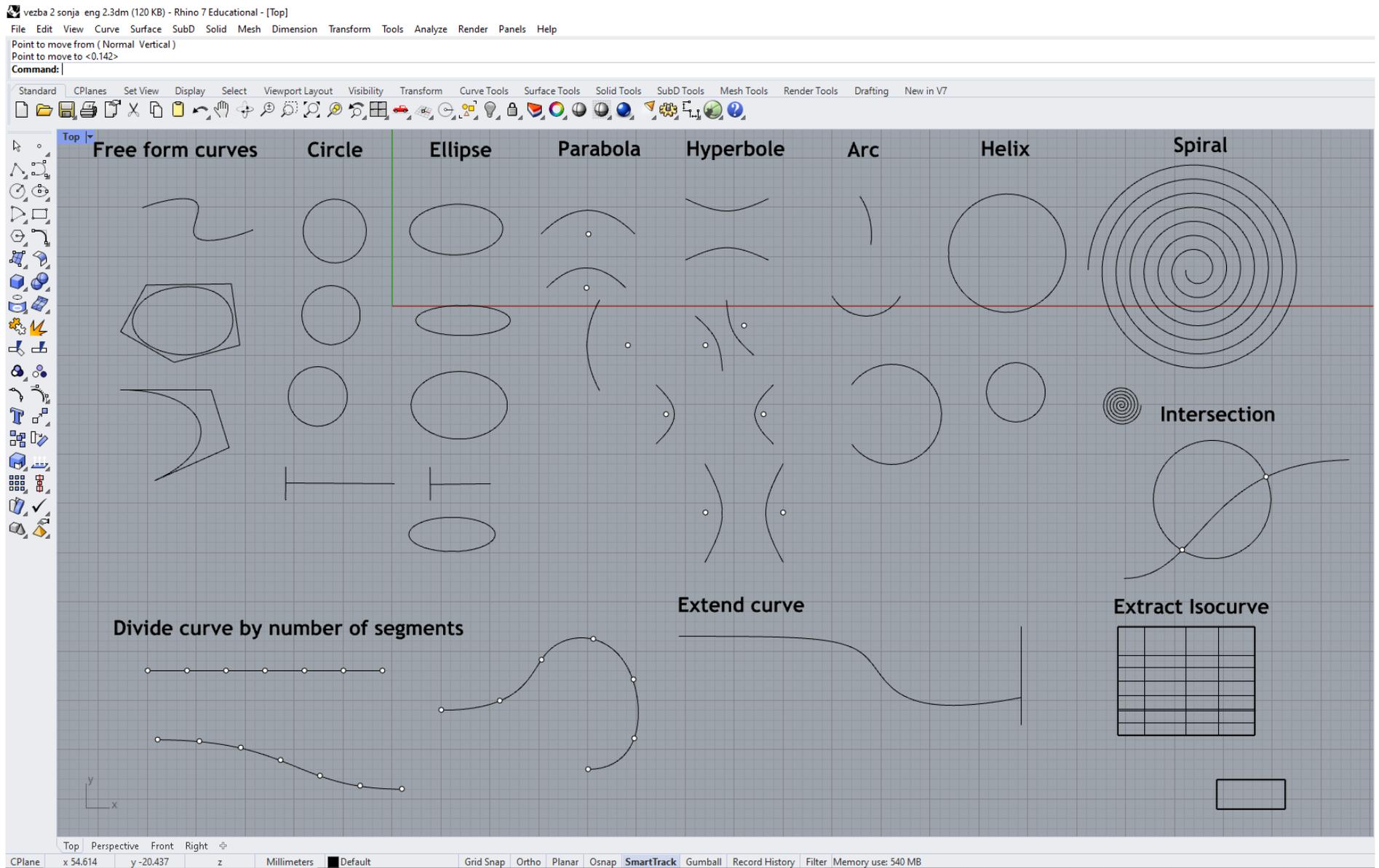


Figure 31 – Example Exercise 2

# UNIT 3 - RHINOCEROS: SURFACES AND GEOMETRIC SOLIDS

## EXERCISE 3

EXERCISE 3 explains the commands for modelling surfaces and solids in the Rhinoceros software. First, the explanation how to construct planes (rectangular surfaces), as the simplest surfaces, is given using three ways most frequently used in practice, found in the main menu (fig. 32). Then it is shown how to use the options **Loft**, **Sweep** and **Revolve**, for modelling surfaces (fig. 33). All previously named commands can be found in mini menu, together with operations that can be performed on surfaces (fig. 34). For these options it is necessary to draw lines or curves as guidelines ( $d_1$ ,  $d_2$ , fig.35) or generatrices, to show how to model surfaces. It is necessary to take into account the order of the steps for modelling surfaces in the command lines (fig. 36 and 37).

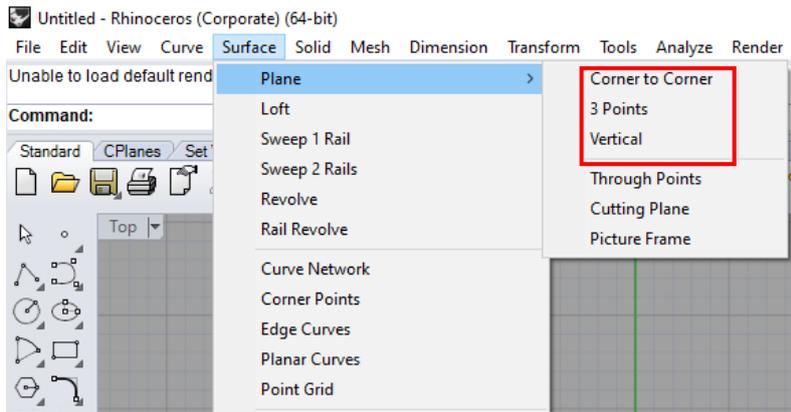


Figure 32 – Surface/Plane

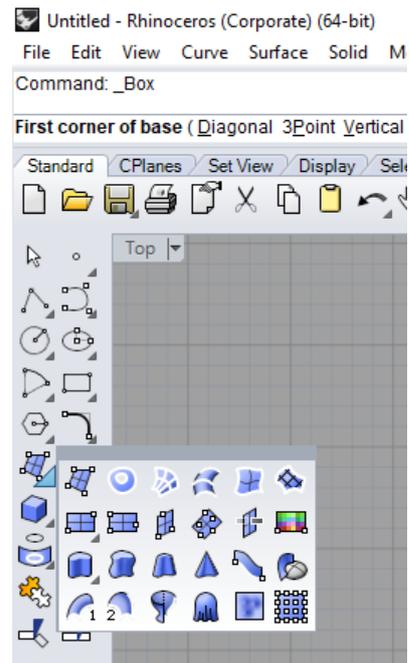


Figure 34 – Surface/Mini Menu

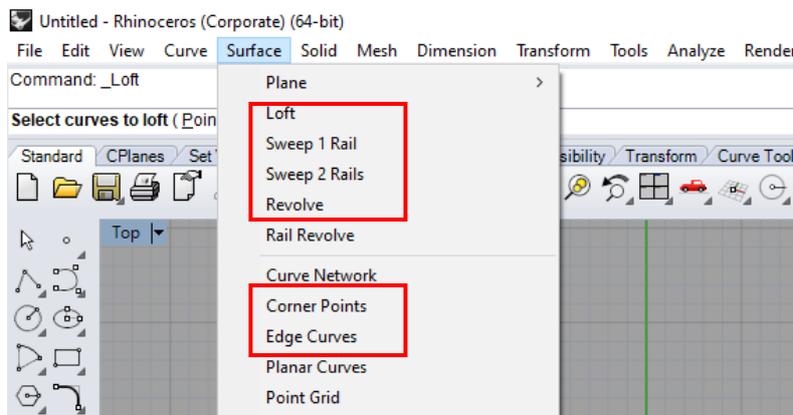


Figure 33 – Surface/Loft/Sweep/Revolve/Corner Points/Edge Curves

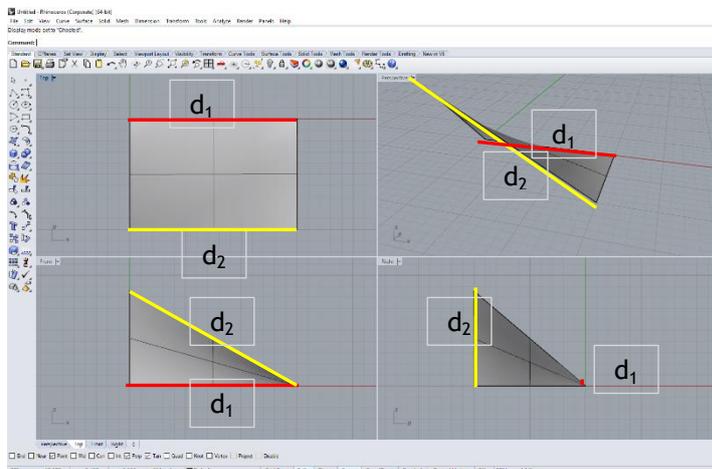


Figure 35 – Option Surface/Loft

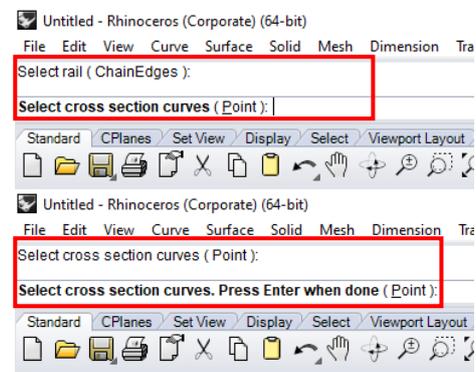


Figure 36 – Surface/Sweep Rail

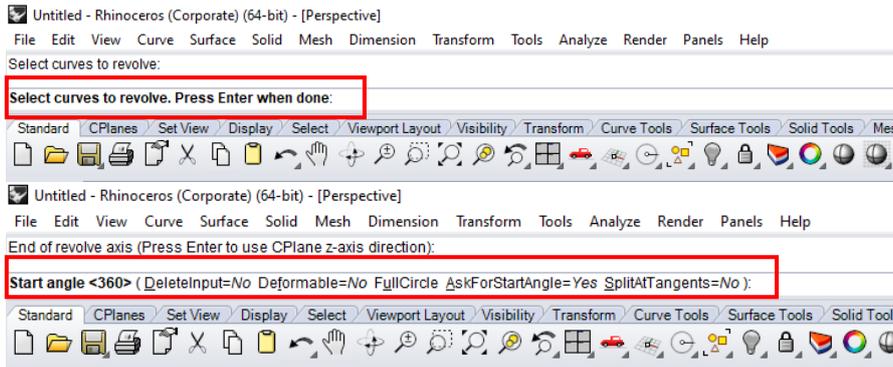


Figure 37 – Option Surface/Revolve/Sequence of steps

Then it is shown how to extend a surface, chamfer it or cut the ends and make an identical surface at the given distance (fig. 38).

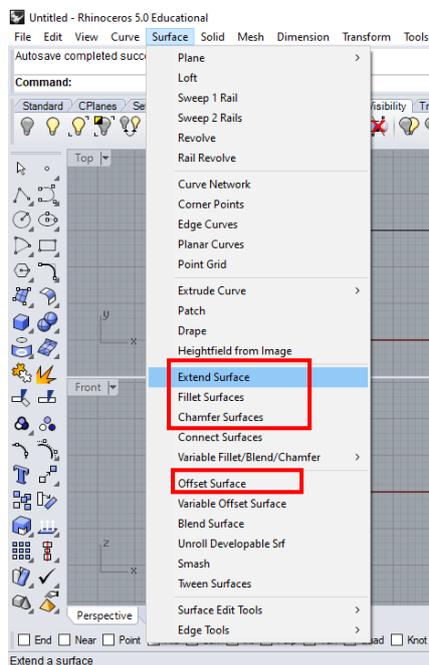


Figure 38 – Extend/Fillet/Chamfer/Offset Surface

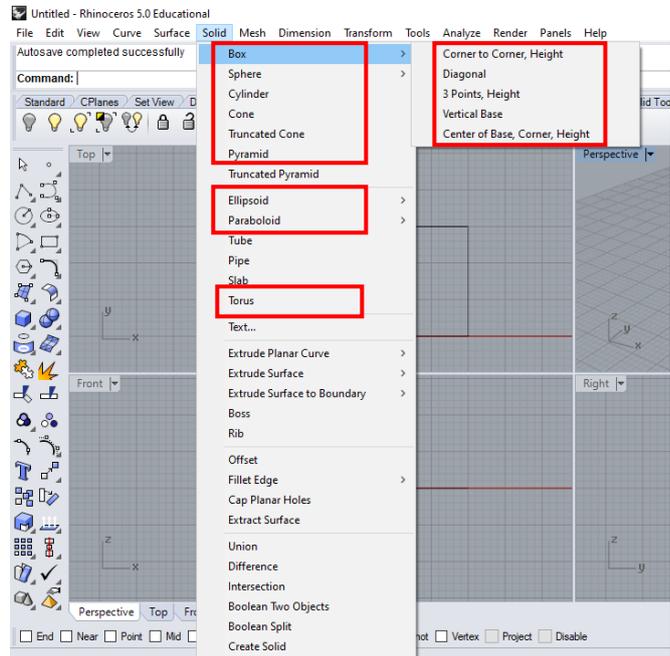


Figure 39 – Solid/Box/Cylinder/Cone/Pyramid...

Then follow the explanations on how to model geometric solids: Box, Sphere, Cylinder, Cone, Pyramid, Ellipsoid, Paraboloid and Torus (fig. 39), if bases and heights are given, as well as centers and radiuses, or axes, or dimensions of the parallels.

Boolean operations of addition, difference and subtraction with geometric solids are shown (fig. 40): how to merge more geometric solids into one whole (**Union**); how to determine the breakthrough of a solid while rejecting one solid from the other (**Difference**); how to determine the core of the breakthrough (**Intersection**); how to reject the core of the breakthrough and join the rest of both solids into one whole (**Boolean two objects**); how to keep both solids while showing the breakthrough line (**Boolean split**). Then it is shown how the tools for transformation of geometric solids can be used (fig. 41): showing all points on the solid, (**Turn on Points – F10**); individual movement of the points (**Edges/Move Edge**); translational movement of the sides of a solid (**Faces/Move Face**).

The visibility of surfaces is also highly important (**Display**), so it is shown how it could be changed into the wire model, full model with the construction or a full model with materialization display (fig. 42, 43 and 44).

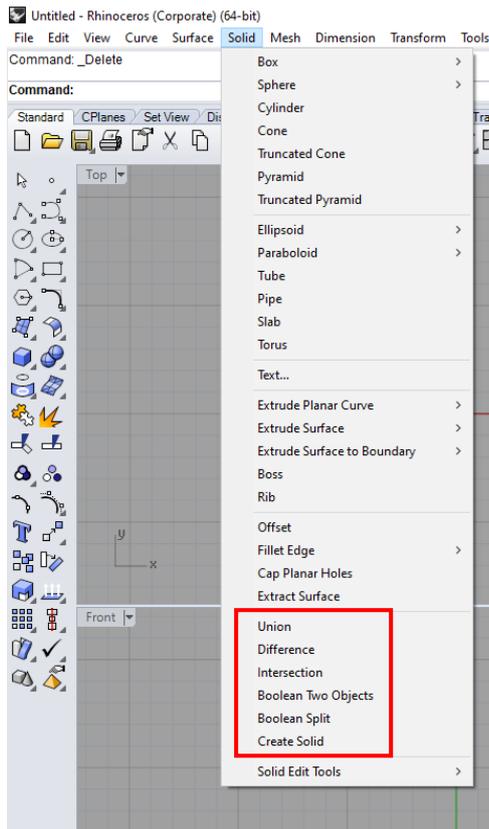


Figure 40 – Operations with Solids

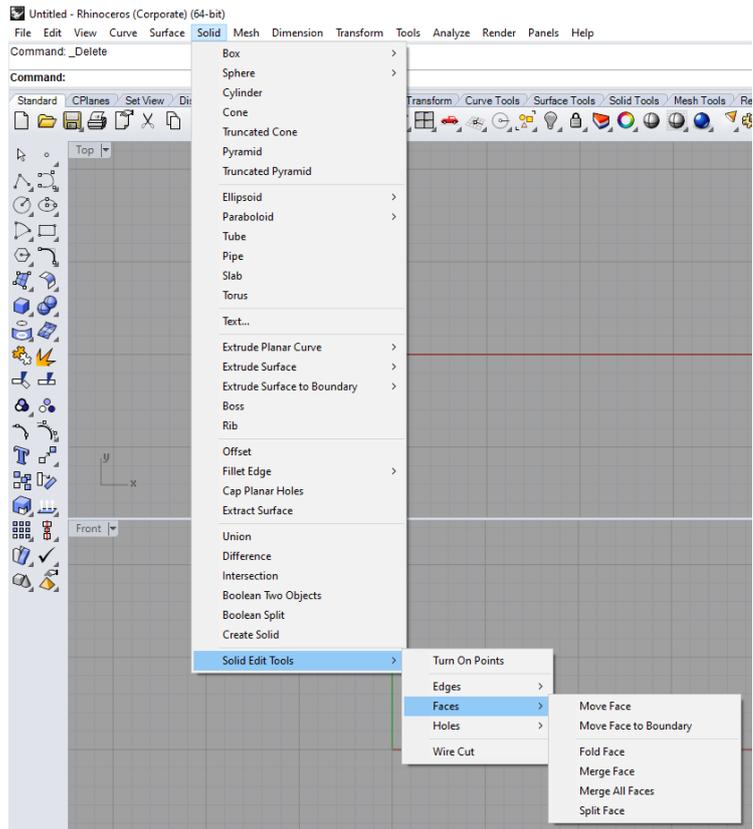


Figure 41 – Solid Edit Tools

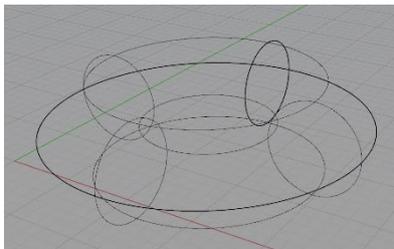


Figure 42 – Wireframe viewport

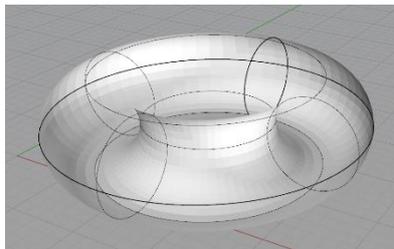


Figure 43 – Shaded viewport

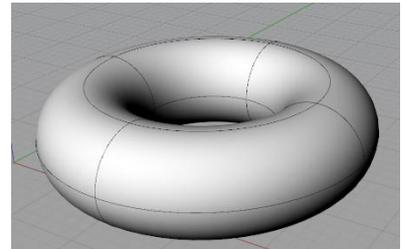


Figure 44 – Rendered viewport

### HOMWORK 3

Homework for Unit 3 is to draw the given meridian, consisting of lines and curves, that merge into one whole using the option **Join** and then model a rotational surface using the option **Surface/Revolve**.

## EXERCISE 3 – Rhinoceros: Modelling surfaces and geometric solids

### Surfaces:

1. **Surface/Plane/Corner to corner** – make rectangular surface by using two opposite angles;
2. **Surface/Plane/3 Points** – make rectangular surface by using 3 points;
3. **Surface/Plane/Vertical** – make vertical rectangular surface;
4. **Surface/Loft** – make a surface of multiple lines/curves;
5. **Surface/Sweep** – make a surface by sweeping profile along rail (curve);
6. **Surface/Revolve** – make surface by rotating line around a vertical axis;
7. **Surface/Corner points** – make surfaces by selecting polygon vertices;
8. **Surface/Edge curves** – create a surface of lines/curves in the same plane;
9. **Surface/Extend surface** – extend surface;
10. **Surface/Fillet surface** – rounded ends;
11. **Surface/Chamfer surface** – cut ends;
12. **Surface/Offset surface** – make surface that is equidistant from all points of the given surface;

### Geometric Solids:

1. **Solid/Box/Corner to Corner, Height** – make solid (prism) by using polygon basis and height;
2. **Solid/Sphere/Center, Radius** – make sphere by using center and radius;
3. **Solid/Cylinder** – make cylinder by using circle basis and height;
4. **Solid/Cone** – make cone by using circle basis and height;
5. **Solid/Pyramid** – make pyramid by using polygon basis and height;
6. **Solid/Ellipsoid** – make ellipsoid with center and length of three axes;
7. **Solid/Torus** – make torus by using largest and smallest parallel;

### Boolean operations with geometric solids:

1. **Solid/Union** – connect more geometric solids into one whole;
2. **Solid/Difference (subtract)** – breakthrough of geometric solids by rejecting one solid from the other;
3. **Solid/Intersection** – core of breakthrough of geometric solids;
4. **Solid/Boolean two objects** – breakthrough of geometric solids with the rejection of the breakthrough core and joining into one whole;
5. **Solid/Boolean Split** – breakthrough of geometric solids keeping both solids and showing breakthrough line;
6. **Solid/Create Solid** – breakthrough of geometric solids while retaining all the cross sections and creating new surfaces;
7. **Solid/Solid Edit Tools/Turn on Points (F10)** – show all points of surface which can be moved individually;
8. **Solid/Solid Edit Tools/Edges/Move Edge** – moving edges of the geometric solid;
9. **Solid/Solid Edit Tools/Faces/Move Face** – translational movement of one surface (face) of the geometric solid;

### Visibility of plotted objects (Display):

- **Display/ Wireframe viewport** – wire model;
- **Display/ Shaded viewport** – full model with construction;
- **Display/ Shade** – full model without construction;
- **Display/ Rendered viewport** – full model showing the materialization;

## HOMEWORK 3: Making a model of a rotational surface

- Draw meridian according to figure 45 in **Front** view. Draw meridian from parts of curves and lines. Using the **Mini Menu/Join** command join the curves and lines into one whole. Use the command **Surface/Revolve** to generate the rotational surface.

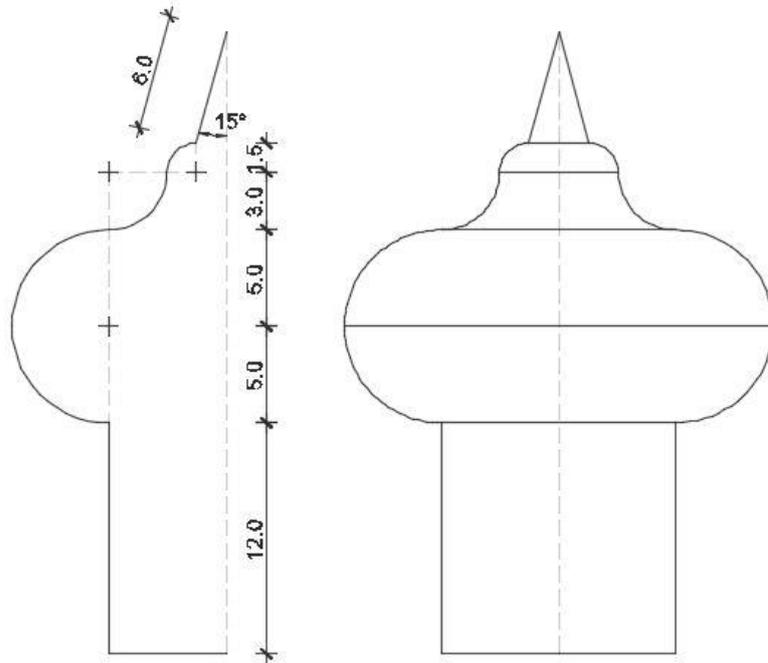


Figure 45 – Meridian of rotational surface

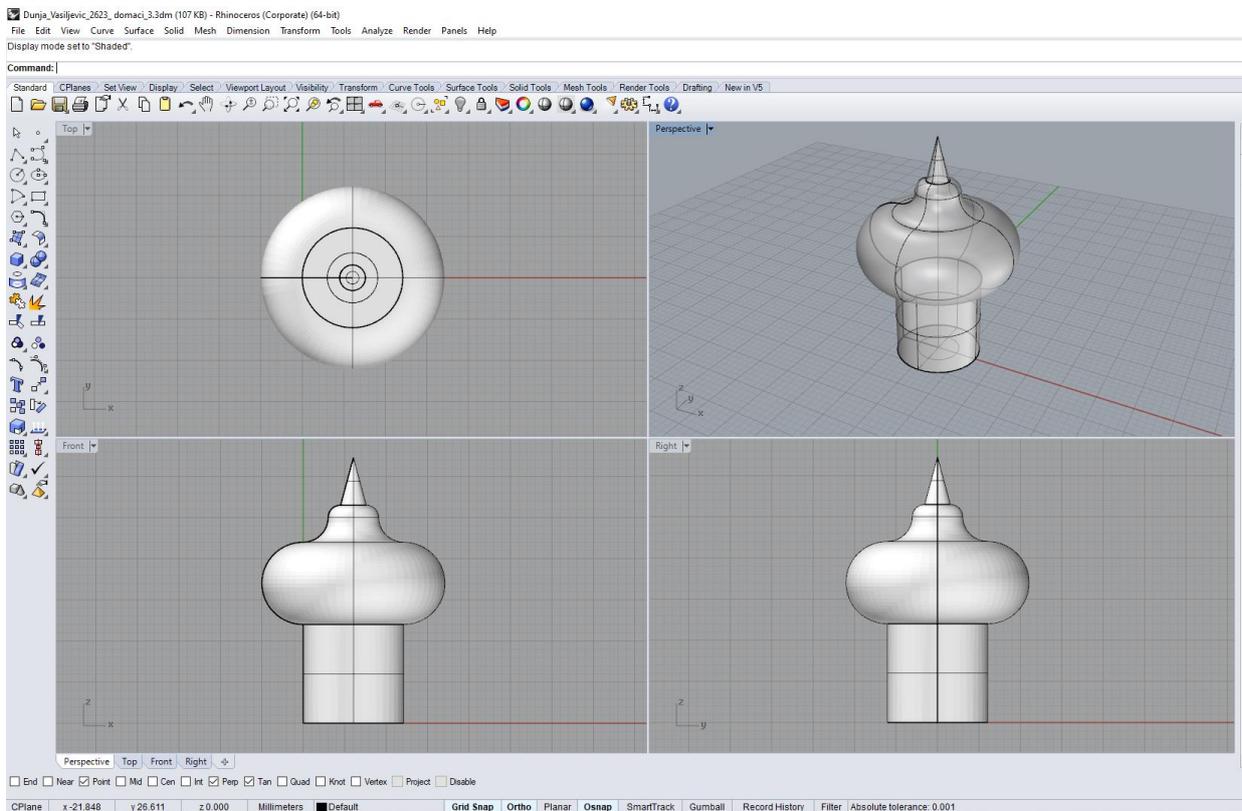


Figure 46 – Example Homework 3

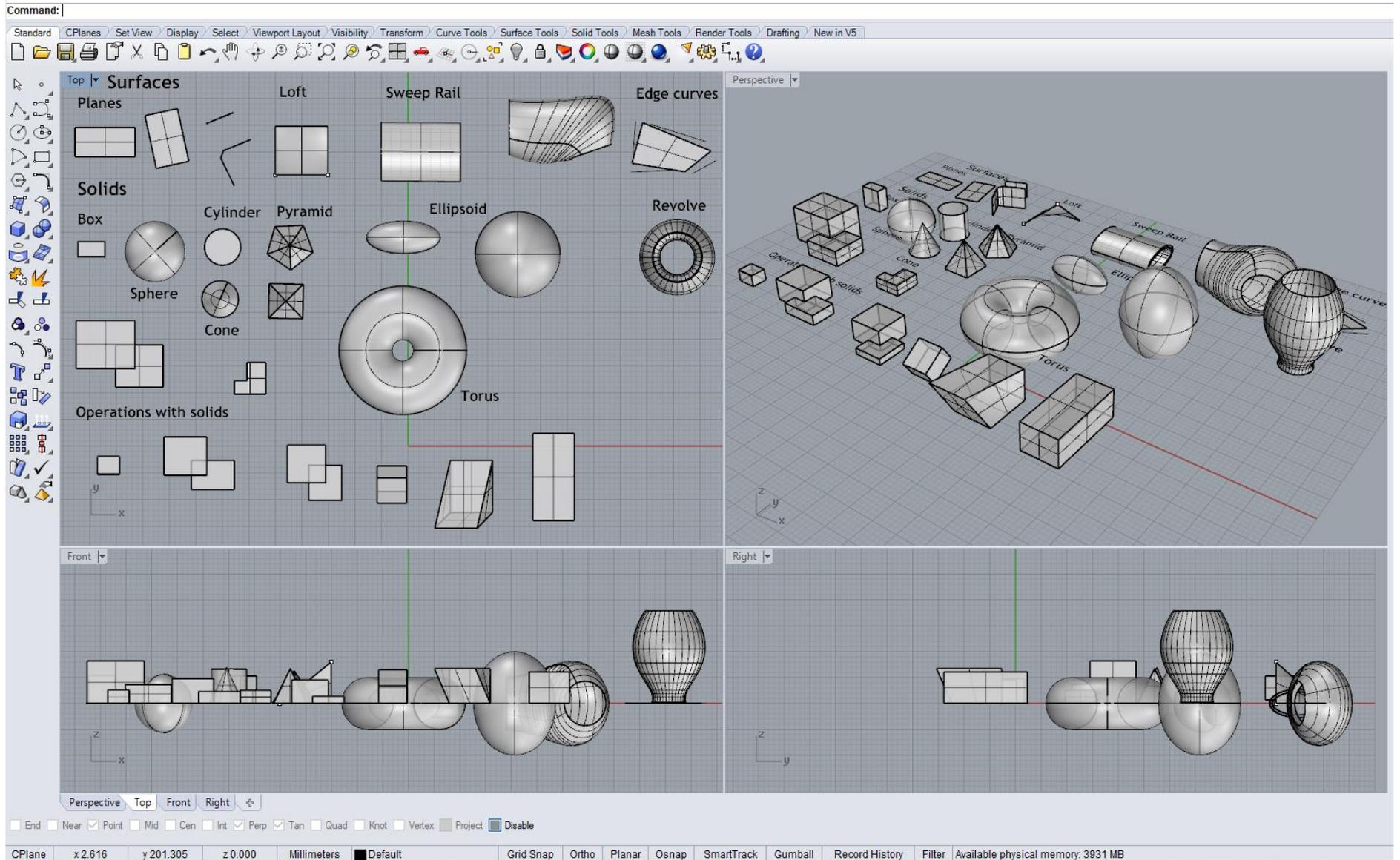


Figure 47 – Example Exercise 3

## UNIT 4 – REGULAR POLYHEDRONS: PENTAGONAL DODECAHEDRON AND ICOSAHEDRON

Polyhedrons are developable, closed surfaces. All faces (parts) of these surfaces are planes (polygons). The polyhedral surfaces include:

- regular polyhedrons (Platonic and Archimedean solids)
- pyramids
- prisms

### REGULAR POLYHEDRONS (PLATONIC SOLIDS)

Regular (primary) polyhedrons or Platonic solids are closed surfaces, in which all faces (parts) are regular polygons – equilateral triangles, squares, or regular pentagons.

Division of regular polyhedrons (Platonic solids):

- Tetrahedron
- Hexahedron (Cube)
- Octahedron
- Pentagonal dodecahedron (PD)
- Icosahedron (ICOS)

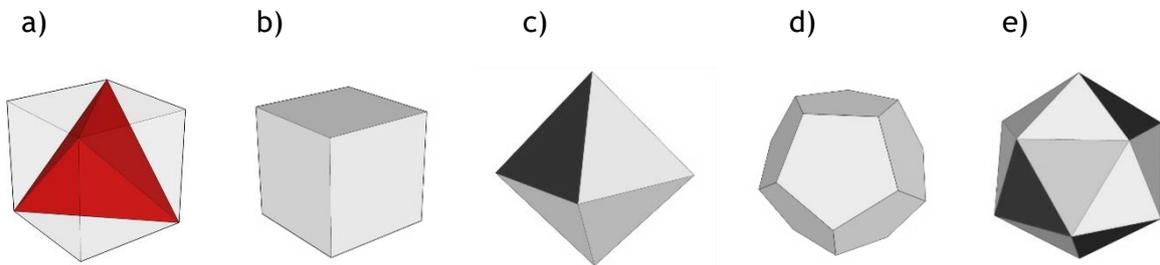


Figure 51 – Regular polyhedrons (Platonic Solids) a) Tetrahedron, b) Hexahedron, c) Octahedron, d) Pentagonal dodecahedron, e) Icosahedron

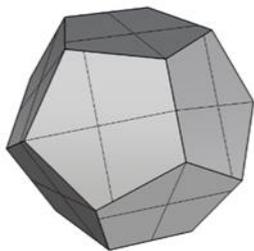


Figure 52 – Pentagonal dodecahedron

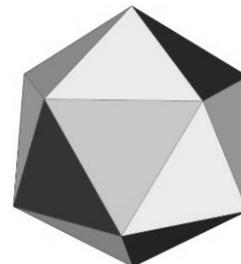


Figure 53 – Icosahedron

Pentagonal dodecahedron – consists of twelve faces (regular pentagons), twenty vertices in which three pentagons and thirty equal edges (sides of the pentagon) converge.

Icosahedron – consists of twenty faces (equilateral triangles), twelve vertices in which five triangles and thirty equal edges (sides of equilateral triangles) converge.

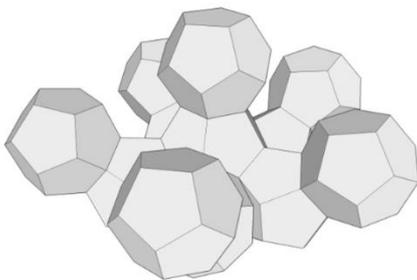


Figure 54 – Possible dodecahedron connections

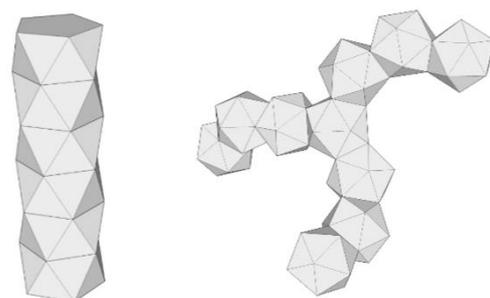


Figure 55 – Possible icosahedron connections

# MODELLING PENTAGONAL DODECAHEDRON AND ICOSAHEDRON IN THE COMPUTER SOFTWARE RHINOCEROS

## EXERCISE 4

EXERCISE 4 shows how to construct pentagonal dodecahedron (fig. 56), one of 5 Platonic solids, in 9 steps. Dimensions of the pentagonal dodecahedron are not specified. In the beginning, one basic pentagon is drawn using the option **Curve/Polygon/Center, Radius**, in **Top** view, and then using the option **Transform/Mirror** the basic pentagon is mirrored 2 times in relation to two sides (fig. 56/1). Then, in **Perspective** view, two circles are drawn in vertical planes like in figure 56/2 and their intersection point is determined. In the same view, using the option **Transform/3d Rotate**, two pentagons are rotated in such a way that one vertex of each of them coincides with the point of intersection of circles (fig. 56/3). The rest of the inclined pentagons are obtained by using the command **Transform/Rotate**, in **Top** view. In that way, the lower half of dodecahedron is generated. The upper half is generated in **Perspective** view, using the option **Orijent3pt**, which is typed on the command line. The order of the choice of points for this command is shown in figure 56/4. Pentagons, that are planar figures in the wire model, transform into planes by using the option **PlanarSrf**, which is typed in the command line, and the **Join** option merges all pentagonal planes into one solid (fig. 56/5).

## HOMEWORK 4

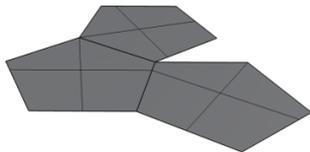
HOMEWORK 4 requires to model icosahedron (fig. 58), another Platonic solid, using 8 steps, dimensions given in meters. That is why it is necessary to change measurement units, from mm to m, using the option **File/Properties/Units**. Icosahedron consists of two equilateral pentagonal pyramids, edge  $a$  (side of equilateral triangle). They are spaced by the height  $H_2 = 0.85a$  and connected by a central part, which consists of equilateral triangles. Using the option **Solid/Pyramid**, the first pentagonal pyramid is constructed, whose side of basis is  $a=10\text{m}$ , and the height  $H_1=5.26\text{m}$ , in **Top** view (fig. 58/1). The base pentagon is constructed using the center and the circumscribed circle around the pentagon, the radius  $r=8.503\text{m}$  (choose number of sides 5). Then the first pyramid is mirrored and copied related to the basis plane in **Front** view (because in this view the mirroring plane is a straight line). Thusly obtained second pyramid is moved for the height  $H_2=8.5\text{m}$ . The second pyramid is then rotated using the option **Transform/Rotate** in **Top** view at the angle of  $36^\circ$  (fig. 58/2). In this way, it is secured that the basis pentagons are rotated at the angle of  $36^\circ$ , so that the intersections of their appropriate vertices are equal edges of icosahedron  $a=10\text{m}$ , i.e. equilateral triangles. In order for the inside of the solid to be empty, it is necessary to separate the wrappers and bases of both pyramids using the option **Explode** and delete the pentagon's basis. The middle part is generated by using the option **Mini Menu/Surface from 3 or 4 other points** (fig. 58/3). The option **Join** merges all planes into one solid (fig. 58/4).

## EXERCISE 4 – Rhinoceros: Model a pentagonal dodecahedron

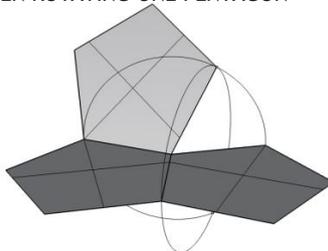
### Procedure for constructing pentagonal dodecahedron:

1. Draw a basic pentagon using the options **Curve/Polygon/Center, Radius** in **Top** view (*it is important that the Number of sides is set to 5*);
2. Using **Transform/Mirror** option, mirror and copy basic pentagon in relation to two of its sides in **Top** view (*Osnap option should be included*) (see sketch 1);
3. In **Perspective** view, draw two circles using option **Curve/Circle/2 Points** (*Vertical option must be included in the command line*) so that one point is on side of one pentagon and the other one belongs to the other pentagon (see sketch 2);
4. Find their intersection point in the same view, using the option **Curve/Curve From Object/Intersection** (see sketch 2);
5. In **Perspective** view, rotate both copied pentagons using the option **Transform/3D Rotate** around the common side with basic pentagon (*so that axis of rotation is a common side*) and rotate the adjacent point of the copied pentagon to the point of intersection of two circles (see sketch 3);
6. In **Top** view, rotate and copy inclined pentagon for each side of the basis of basic pentagon (*center of rotation – center of basic pentagon*) using option **Transform/Rotate**;
7. Using **Orijent3pt** option (*on command line*) choose between three points on a wire model to make it coincide with three points upon rotation and copying (see sketch 4);
8. Convert wire model into surface by selecting whole wire model and typing option **PlanarSrf**;
9. In order for all surfaces to become one solid, select all surfaces with the **SetSrf** option, and then join them into one whole using the option **Join**;

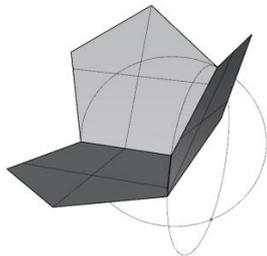
1. DRAWING THREE PENTAGONS WITH ONE SAME SIDE BETWEEN THEM



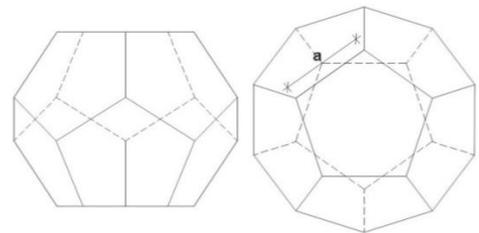
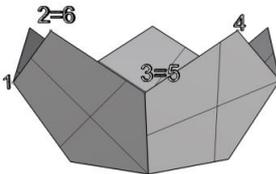
2. DRAWING FIRST TWO VERTICAL CIRCLES, THEN ROTATING ONE PENTAGON



3. ROTATING SECOND PENTAGON TO POINT OF INTERSECTION OF TWO CIRCLES



4. ORDER OF MARKING POINTS WHILE USING OPTION "ORIJENT3pt"



5. FINAL MODEL

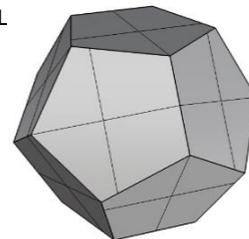


Figure 56 – Procedure for modelling pentagonal dodecahedron (left) and top and front view (right)

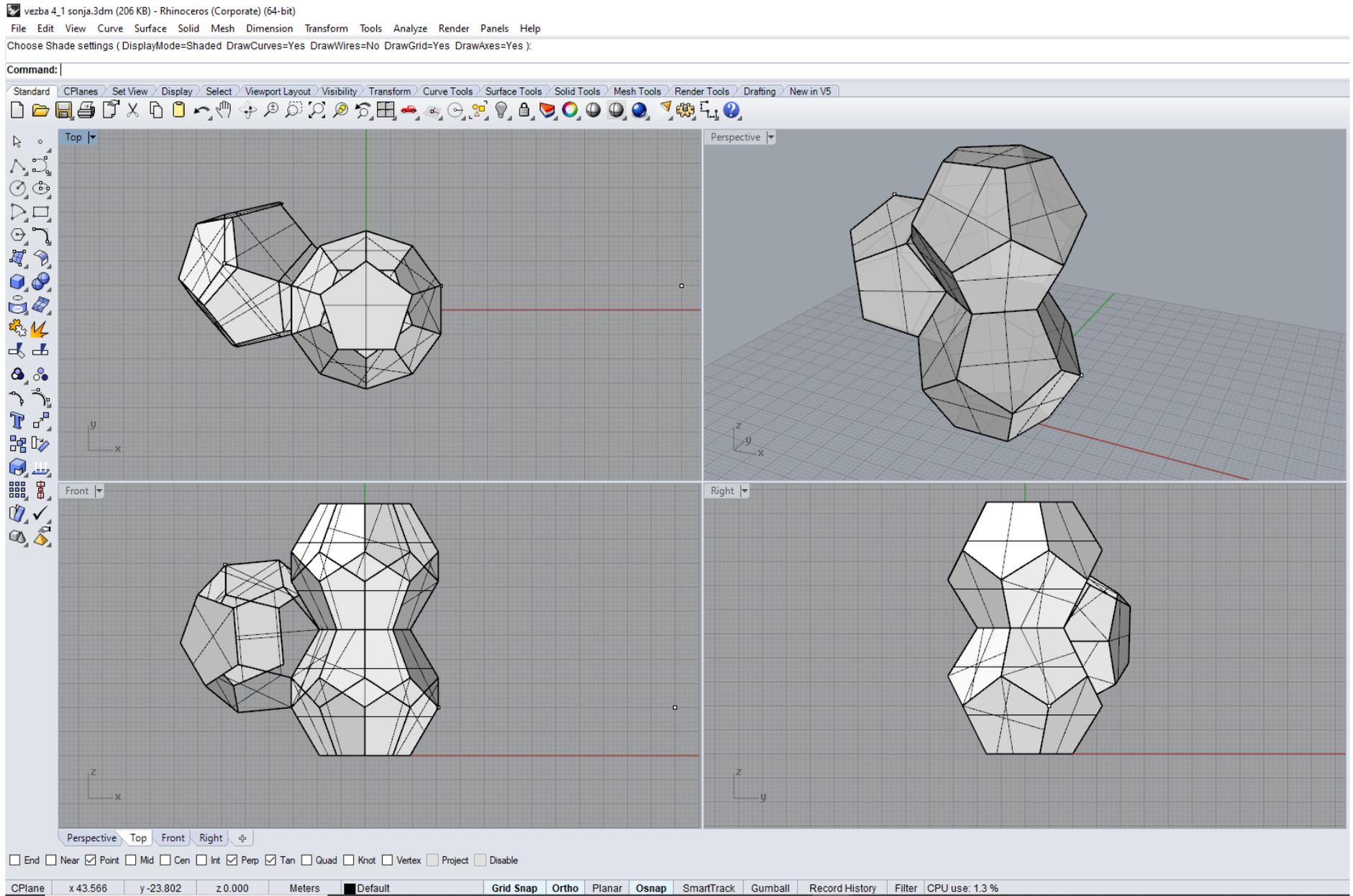


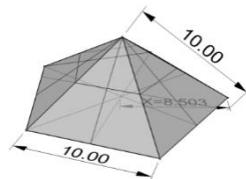
Figure 57 – Example Exercise 4

## HOMEWORK 4: Model an icosahedron

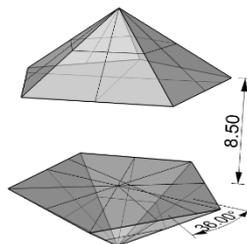
### Procedure for constructing icosahedron from figure 58:

1. In **Perspective** view use the option **Pyramid** from **Solid** card to construct pyramid with the radius of described circle  $r=x=8.503m$  around pentagonal basis and height is  $H_1=5.26m$ ;
2. Using **Mirror** option in **Transform** card in **Front** view, copy and mirror pyramid (*mirroring plane is pentagonal pyramid basis, it is important to include **Copy = Yes** option to get a copy of pyramid*);
3. Rotate upper pyramid by **Rotate** option for angle of  $36^\circ$  in **Transform** card in **Top** view (*center of rotation – center of pentagon*);
4. In **Front** view using the option **Transform/Move**, put down lower pyramid for height  $H_2 = 8.50m$ ;
5. Using **Explode** option in **Perspective** view, separate two pyramids into separate surfaces, and then by **Delete** option delete both pentagons;
6. In **Perspective** view using the option **Mini Menu/Surface** from **3** or **4** other points, create equilateral triangles that close the icosahedron;
7. Using the **Join** option in **Perspective** view connect all surfaces into one solid;
8. Length of the icosahedron's side is the triangle side  $a=10m$ , through which is expressed radius of described circle  $x=0.8503a$  and height  $H_2=0.85a$  of icosahedron;

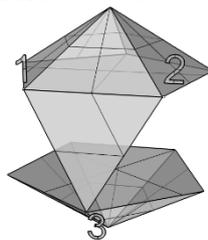
1. DRAWING 5 SIDED PYRAMID WHERE  $X= 8.503M$  AND  $H1= 5.26M$



2. MIRROR, ROTATE AT  $36^\circ$  AND MOVE DOWN FOR 8.50M SECOND PYRAMID



3. DRAWING ONE EQUILATERAL TRIANGLE WHICH CONNECTS TWO PYRAMIDS USING POINTS 1, 2 AND 3



4. FINAL MODEL

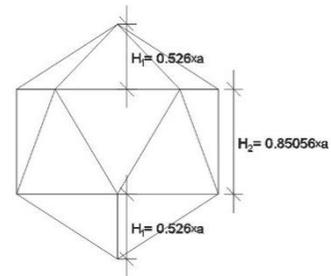
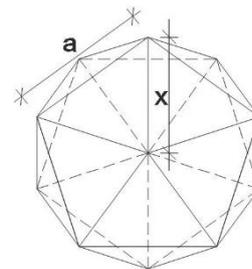
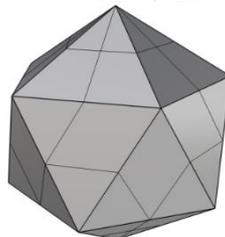


Figure 58 – Procedure for modelling icosahedron (left) and top and front view (right)

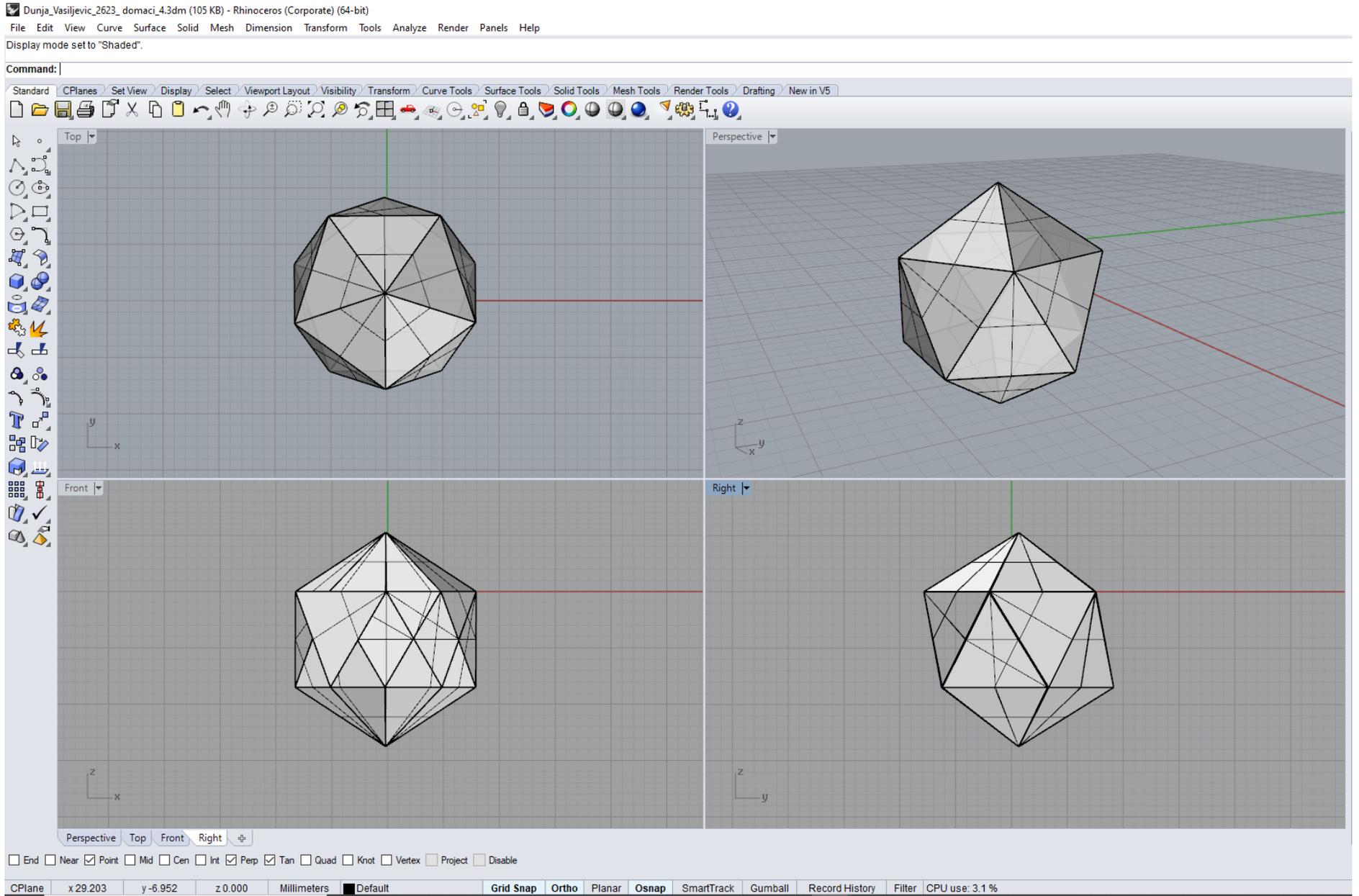


Figure 59 – Example Homework 4

Figures 60 and 61 show representative examples of the use of pentagonal dodecahedron and icosahedron in architecture.



Figure 60 – PD, Ramot Housing, Jerusalem, Israel, arch. Zvi Hecker, year 1971-1985



Figure 61 – ICOS, Village Inc, arch. Jill Fehrenbacher, year 2005

## UNIT 5 – POLYHEDRAL SURFACES: PYRAMIDS AND PRISMS

The pyramid and prism belong to polyhedral open surfaces, because they can extend to infinity. Both surfaces may be limited by a plane to generate a closed surface. Surfaces which are generated by central extrusion of polygon are pyramids and those generated by parallel extrusion of polygon are prisms. These surfaces belong to ruled surfaces too.

Pyramid (fig. 62) is generated if a line which passes through a constant point  $S$  is sliding on the polygon. A polygon can be planar or spatial, open or closed, with unlimited number of sides and it is a **directrix**  $d$  of surface. Point  $S$  is the top of the pyramid, and couplings of any point on polygon with the top of the pyramid are **generatrices**  $g$  of pyramid.

If the top  $S$  is infinitely distant,  $S_\infty$ , **generatrices**  $g$  are parallel to each other and such a surface cut by two planes is called prism (fig. 63). Those two planes are identical polygons which are **directrices**  $d$  of prisms.

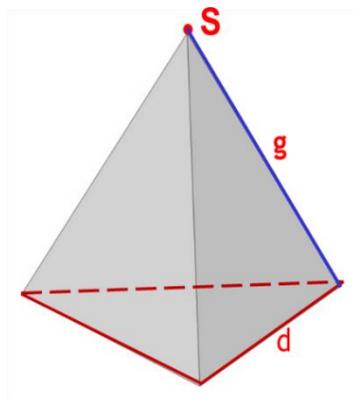


Figure 62 – Pyramid

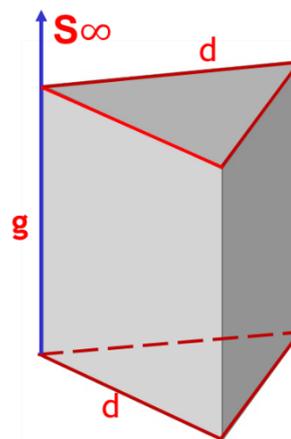


Figure 63 – Prism

Pyramids and prisms belong to ordinary polyhedrons, because they are made of planar faces, which may be regular or irregular triangles or quadrilaterals (wrapper) and the basis which can be regular or irregular polygon. Pyramids are usually determined by basis and top, and prisms with two parallel bases.

## FOLDS STRUCTURES

In contemporary architecture, pyramids and prisms are used in folds structures (fig. 64 and 65), because they are the simplest geometric forms, which can be used for spatial carriers. Spatial carriers are composed from parts of pyramidal or prismatic shape, or a combination of both forms.

Since a part of the pyramids and prisms can develop into a plane, corresponding to rotations around the edges, these spatial carriers become support, if stiffening bearings ensure the immutability of the form folds. Besides, many folds structures were created by breaking through pyramids and prisms, because of better mutual stiffening.

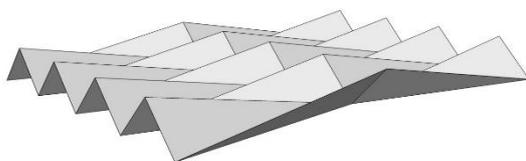


Figure 64 – Example of Pyramidal folds

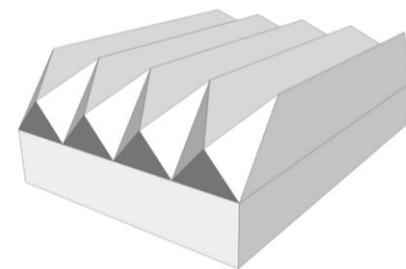


Figure 65 – Example of Prismatic and Pyramidal folds

## MODELLING PYRAMIDAL AND PRISMATIC FOLDS IN THE COMPUTER SOFTWARE RHINOCEROS

### EXERCISE 5

EXERCISE 5 shows how to construct pyramidal folds over a circular basis (fig. 66), in 10 steps, given dimensions in meters. That is why it is necessary to change measurement units, from mm to m, using the option **File/Properties/Units**. Firstly, one element of pyramidal fold is formed. It consists of one horizontal normal and one reversed inclined pyramid that have the common basis in vertical plane. The top of the horizontal pyramid is in the center of the circular basis of the object, and the top of the inclined, reversed pyramid is on the bigger circle of the object. In order to construct one element of the fold (two pyramids), the options used from the main menu were **Curve/Circle** and **Divide Curve by Number of Segments**. Firstly, two concentric circles are drawn, with radius  $r=13\text{m}$  (fig. 66/1) and  $r=23\text{m}$  (basis of the object fig. 66/2 and 66/5), and then divided into equal number of parts, smaller one into 50, and the bigger one into 25 (the division should start from one axis in the window). The points on both circles, obtained by this division, are marked, as well as the common center. Using the option **Move**, the bigger circle is moved translationally for 10m (fig. 66/2) downwards ( $-z$  coordinate), and one point on the smaller circle (it is best if it is on one axis in the window), for 2 m upwards ( $+z$  coordinate). This was necessary to separate the points forming one element of the fold (two pyramids). One half of the elements of the fold is modelled in **Perspective** view, using the command **Mini Menu/Surface from 3 or 4 Other Points**, in such a way as to merge appropriate points on both circles and the common center, in order to get one surface of the mantle of the horizontal pyramid, and then one surface of the inclined pyramid. (fig. 66/3). In order to obtain the second half of the elements of the fold, it is best to use the option **Mirror**, in **Top** view (fig. 66/4). All separate surfaces of one element of pyramidal fold are merged into one whole using the option **Join**. Then, using the option **Transform/Array/Polar**, one element of the fold is radially copied 25 times, in order to obtain the whole object (fig. 66/5). The option **Transform/Array/Polar** is very convenient for this kind of object, because it performs rotation and copying at the same time. The center should be the common center for both circles, the number of copying 25, and the rotation angle  $360^\circ$ .

### HOMEWORK 5

HOMEWORK 5 shows how to construct prismatic and pyramidal fold (fig. 68), in 8 steps, given dimensions in meters. That is why it is necessary to change measurement units from mm to m using the option **File/Properties/Units**. Firstly, one element of the fold is formed, consisting of one horizontal prism (roof) and two symmetrical reversed vertical pyramids (columns), with common bases in the inclined planes. One half of the shell of prism surface is formed from a rectangle (fig. 68/1), which was drawn using the option **Curve/Rectangle**, and then, using the command **F10**, points of the rectangle were separated, and two were moved using the option **Move**, in two directions ( $x$  and  $z$ ), in order to obtain an inclined trapeze (fig. 68/2 and 68/3). The surface of the prism shell was obtained using the command **Mini Menu/Surface from 3 or 4 other points**, selecting the 4 vertices of the trapeze. Then, it was mirrored using the option **Mirror** in relation to the longer side of the trapeze (fig. 68/3). The top of the pyramidal fold was obtained using the point which was on the longer axis of the prism symmetry lowered for 7 m in relation to the lower generatrices of the prism and obtained using the option **Polyline** (fig. 68/4). The surface of the half of the shell of the reversed pyramid was obtained using the option **Mini Menu/Surface from 3 or 4 other points**, by merging two appropriate points of the prism basis with the pyramid top and was mirrored in **Top** view in relation to the longer symmetry axis of prismatic fold, in order to obtain a whole column. Using the option **Join**, the column was merged into one whole, and then mirrored in relation to the shorter axis of the roof symmetry, best in **Front** view (fig. 68/5). All surfaces of the folds of prism and pyramid were merged into one whole using the option **Join**, and then copied three more times (fig. 68/5).

## EXERCISE 5 – Rhinoceros: Model the given object using pyramid

### Procedure for constructing pyramidal fold structure from figure 66:

1. Draw circle of radius  $r=13\text{m}$  using the option **Curve/Circle/Center, Radius** in **Top** view;
2. Then, in the same view, divide circle into 50 equal parts using the option **Curve/Point Object/Divide Curve by Number of Segments** (before setting the number of division choose **Natural** in the command line, then **Mark ends=Yes**);
3. Mark circle center using the **Point** option;
4. Raise one point from circle for 2m using the option **Move** in **Perspective** view (include **vertical = yes**);
5. In **Top** view, draw a circle with the same center as first, and radius  $r=23\text{m}$ ;
6. Put down the larger circle for 10 m using the option **Move** (precisely in **Front** view);
7. Then divide the circle  $r=23\text{m}$  into 25 equal parts in **Top** view using the option **Curve/Point Object/Divide Curve by Number of Segments**;
8. In **Perspective** view using the option **Surface from 3 or 4 other points**, connect two adjacent points to the center of smaller circle as well as to point of larger circle, make two surfaces (see sketches 3 and 4);
9. Select all obtained polygons and using the option **Join** merge them into one unit;
10. In **Top** view, rotate and copy the basic element across the circle with **Transform/Array/Polar** option, 25 times for an angle of  $360^\circ$ ;

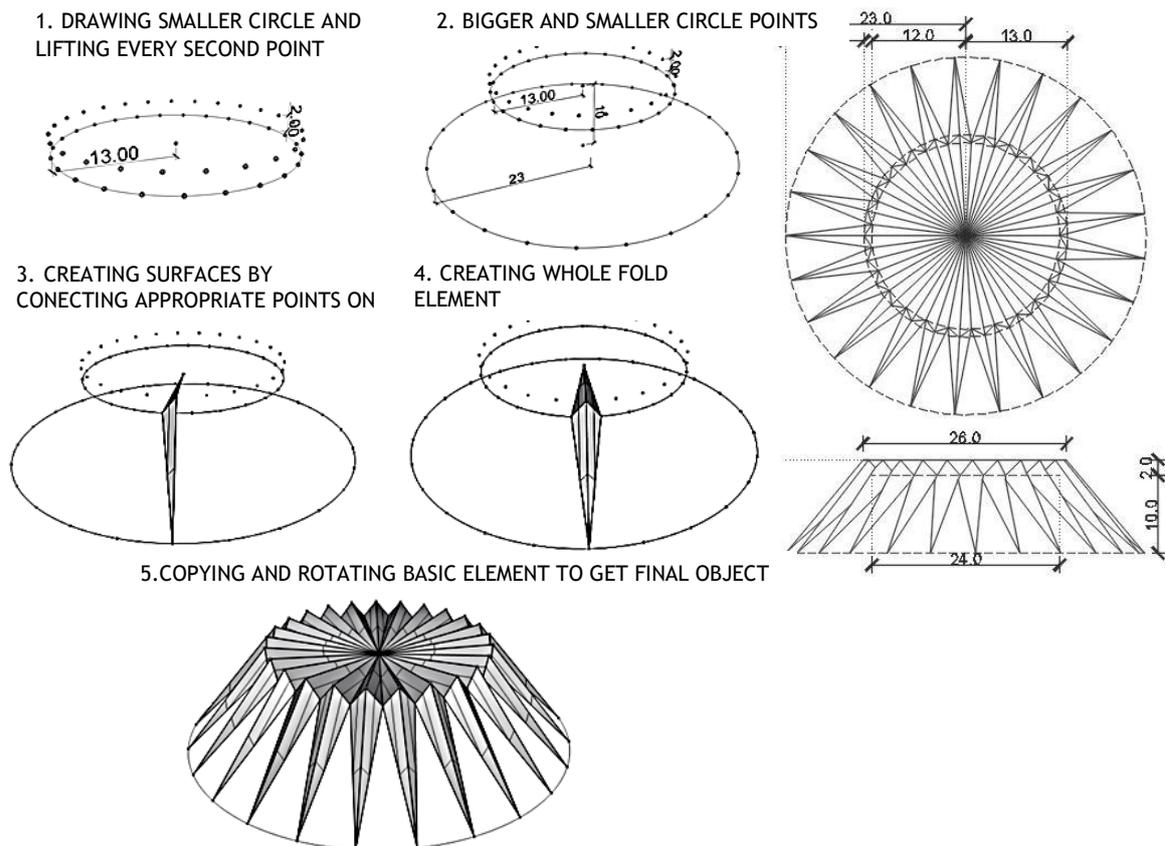


Figure 66 – Procedure for modelling pyramidal folds (left) and top and front view (right)

vezba 5 sonja.3dm (457 KB) - Rhinoceros (Corporate) (64-bit)

File Edit View Curve Surface Solid Mesh Dimension Transform Tools Analyze Render Panels Help

Display mode set to "Ghosted".

Command:

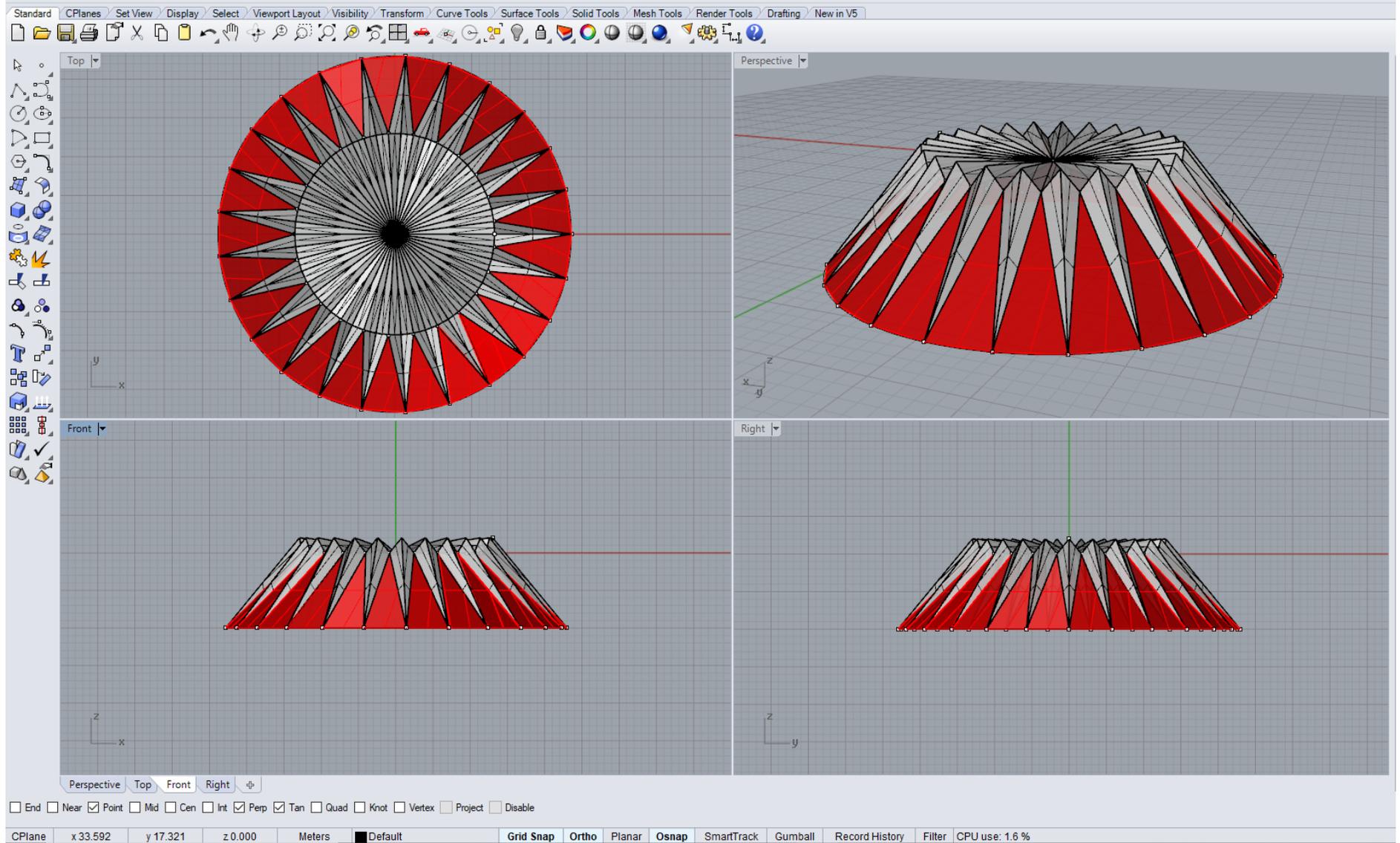


Figure 67 – Example Exercise 5

## HOMEWORK 5: Model the given object using prism and pyramid

Procedure for constructing prismatic and pyramidal fold structure from figure 68:

1. Draw a rectangle in **Top** view using **Curve/Rectangle/Corner to angle** option, measuring 10x1m;
2. Using option **F10**, give the rectangle points to manipulate and move using the option **Move** two adjacent points on the long side in the x and z direction for 1.5m (*symmetrically one and the other point*);
3. Draw surface using the option **Mini Menu/Surface from 3 or 4 Corner Points** connecting all 4 rectangle (trapeze) points in **Perspective** view;
4. Using option **Transform/Mirror** copy surface symmetrically relative to long side of trapeze, in **Top** view;
5. In **Front** view, draw the vertical **Polyline** height 7m from the lower edges roof structure so that the line is located on axis of symmetry (long side of the roof structure – see sketch 4);
6. Using the option **Mini Menu/Surface from 3 or 4 Corner Points**, draw a 3-point surface pillar (top two points on trapeze and bottom point on the vertical line – see sketch 5). Draw the pillar so that it is on both sides of the roof (*it can be obtained with Mirror option in Front view, if Polyline is not drawn on both sides*);
7. Using the option **Join** connect all surfaces into one;
8. Copy three more such units one behind the other using the option **Transform/Copy** (see sketch 6 on fig. 68);

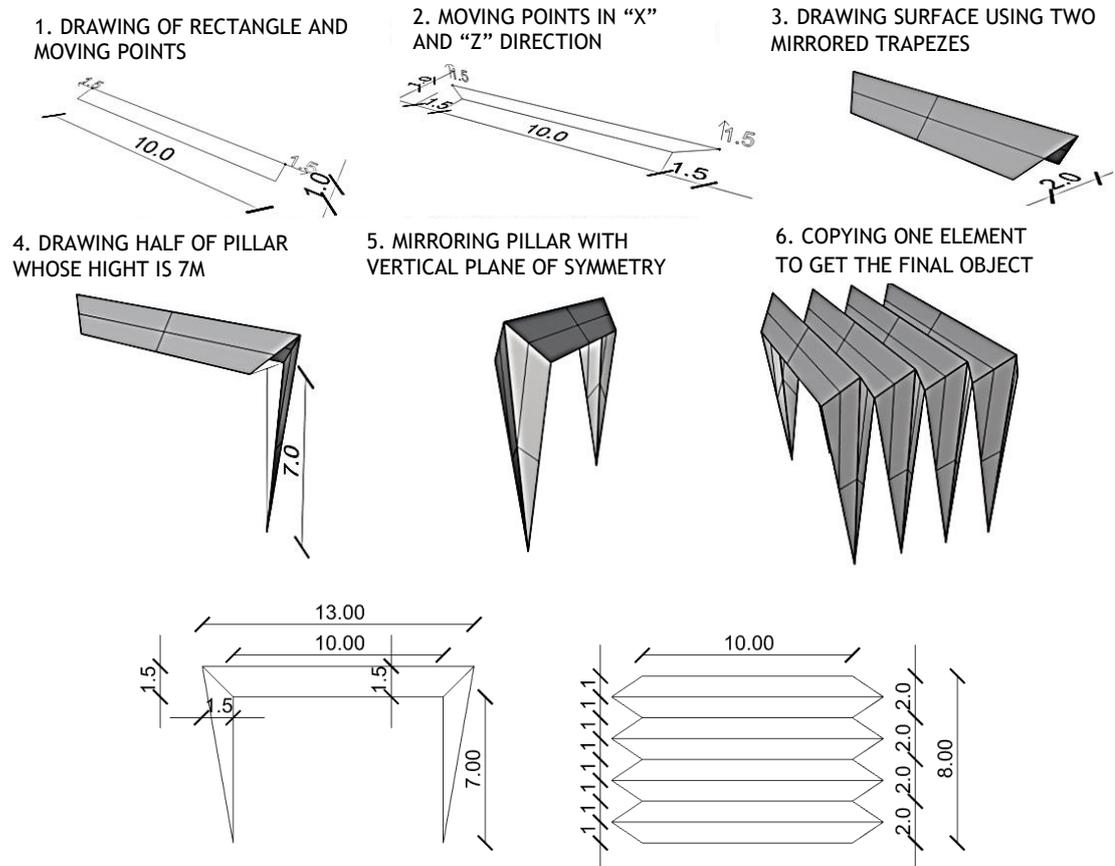


Figure 68 – Procedure for modelling prismatic and pyramidal folds (up) and top and front view (down)

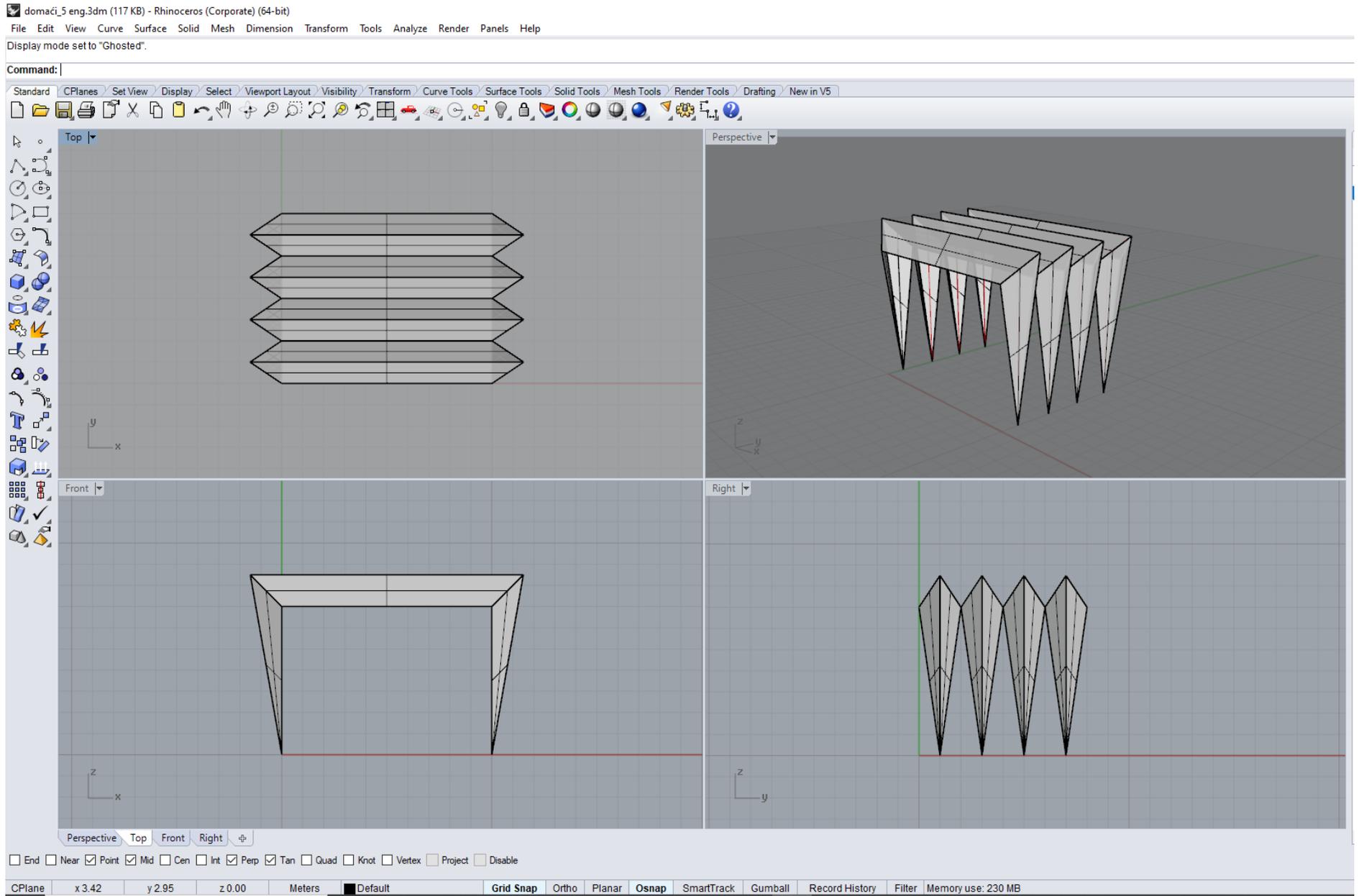


Figure 69 – Example Homework 5

Figures 70 and 71 show representative examples of the use of pyramidal and prism's folds in architecture.



Figure 70 – Pyramidal folds, Canadian Pavilion, World Expo 2010, China, year 2010



Figure 71 – Pyramidal and prism's folds, Air Force Academy Chapel, USA, arch. W. Netsch, year 1963

## UNIT 6 – EXTRUSION SURFACES: CONES AND CYLINDERS

Surfaces which are generated by central extrusion of smooth curve are cones and those generated by parallel extrusion of smooth curve are cylinders. These surfaces can also be generated by movement of straight line **generatrix** around curve **directrix** that classifies them as ruled developable surfaces, which have a single curvature. If that movement is rotation, these surfaces are rotational.

In order to develop the surface into a plane, **generatrix** of surface as it moves, passes with one point outside the curve, the top in finiteness  $S$  (cone), or at infinity  $S^\infty$  (cylinder). It follows that the single-curved surface includes all types of cones (coupe, fig. 72) and cylinders (rollers, fig. 73).

A **directrix** (guideline)  $d$ , by which the **generatrices**  $g$  are moving can be planar or spatial curve, open or closed. If the **directrix** (guideline) is planar curve of 2nd order, then the surface (cone or cylinder) is 2nd order. A **directrix** (guideline) is a basis of cones and cylinders.

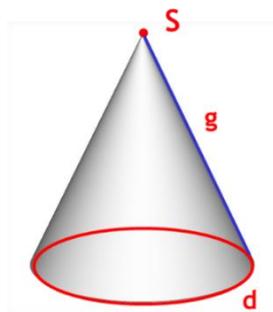


Figure 72 – Cone

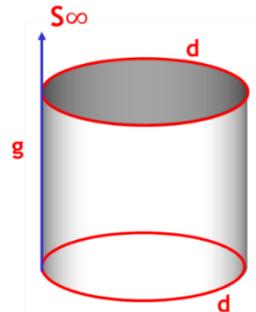


Figure 73 – Cylinder

**Different division of cones and cylinders according to basis (fig. 74):**

- circular (basis is a circle, cones and cylinders,)
- elliptical (basis is an ellipse, cylinders, fig. 74/1)
- parabolic (basis is a parabola, cylinders, fig. 74/2)
- hyperbolic (basis is a hyperbole, cylinders, fig. 74/3)



Figure 74 – Cylinders 1 2 3

### FOLDS STRUCTURES

The simplest curved surfaces are **cylinders** and **cones**. Only limited conical and cylindrical surfaces are used in architectural practice. Single-curved shells are easy to lose shape transverse to the generatrices, but they must be stiffened by compartments and ribs.

Mutual breakthroughs of cones and cylinders stiffen the construction of the object, as well as the breakthroughs of pyramids and prisms. The long cylindrical shell operates as a beam with a curved cross-section, and brief shells bear the load as the vault. Conical shells can be put together like a dome (fig. 75) or repeated opposite each other for covering industrial halls. Cylindrical shells can use whole or parts of cylinders to cover objects (fig.76).

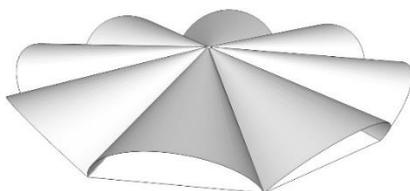


Figure 75 – Example of conical folds

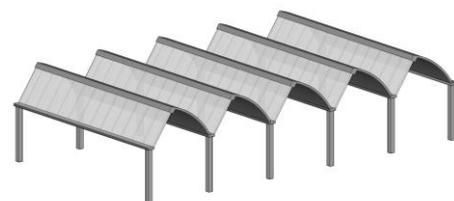


Figure 76 – Example of cylindrical folds

# MODELLING CONICAL AND CYLINDRICAL FOLDS IN THE COMPUTER SOFTWARE RHINOCEROS

## EXERCISE 6

EXERCISE 6 shows how to construct conical folds over the circular basis (fig. 77), in 11 steps, dimensions given in meters. That is why it is necessary to change measurement units, from mm to m, using the option **File/Properties/Units**. In order to model a cone, which is the basic element of the fold, the option **Solid/Cone** is used. Rotational cone is chosen, whose basis lies in the frontal plane, radius  $r=5\text{m}$ , and the axis is horizontal line, length 20m, which is determined in **Right** view (fig. 77/1). Cone is then rotated, using the option **Transform/Rotate**, so that the axis stands at the angle of  $-25^\circ$  in relation to the starting position (center of the rotation is the center of the basis, fig. 77/2), in the same view. In order to obtain the whole object, the basic cone is radially copied using the option **Transform/Array/Polar**, 12 times at the angle of  $360^\circ$ , whereby the cones overlap. Since only the upper part of the conical wrapper is used for the object, it is necessary to fix the inside of the object and that is why cross sectional planes were placed, both horizontal and vertical. The whole object is very precisely cut using the option **Osnap/Intersection** firstly with one horizontal plane, (set through the upper points of basis intersection, fig. 77/3), which is best done in **Front** view, and the plane itself and the part under it are rejected. Then, the conical folds was cut with 6 vertical planes, passing through the center of the object, and they were set precisely in **Top** view. Then comes the cutting of conic folds, using radial vertical planes, using the **Split** option, and the removal of all surfaces on the inside of the object as well as vertical intersecting planes. In order to make the conic fold a hollow surface, it was divided into parts (shells and bases) using the option **Mini Menu/Explode** and the bases were removed, using the option **Delete**. It is necessary to close the object with a vertical cylinder, which is why a circle was constructed written into a basic dodecagon, using the option **Curve/Circle**, the shell was elevated using **Surface/Extrude Curve/Straight** (fig. 77/4) and using **Trim**, the excess of the cylinder was removed. The thickness of the shell was obtained using the command **Solid/Offset**. The option chosen was to give the thickness of the entire conical fold over the existing shell (fig. 77/5).

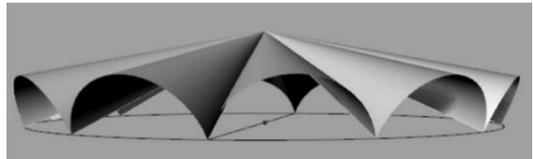
## HOMEWORK 6

HOMEWORK 6 is to model a vault, obtained by breakthrough of three cylinders over the basis which is an equilateral triangle (figure 79), in 8 steps, with given dimensions. That is why it is necessary to change measurement units, from mm to m, using the option **File/Properties/Units**. Firstly, an equilateral triangle was constructed, side  $a=20\text{m}$ , using the option **Curve/Polygon** and its orthocentre was determined (fig. 79/1). Over one side of the triangle, as a diameter of circular bases, a horizontal cylinder was constructed, using the option **Solid/Cylinder**. In **Front** view, the basis of the cylinder was constructed, and in **Top** view, the length to the triangle orthocentre was determined (fig. 79/2). The cylinder was cut with two vertical planes, through the height of the triangle, and one horizontal plane, through the basis center (fig. 79/3). Parts of the cylinder were removed, under the horizontal plane and outside of the triangle up to the intersection with vertical planes (fig. 79/4). The basic element of the cylindrical vault was then radially copied 3 times using the command **Transform/Array/Polar**, the center of the copying is the triangle orthocentre, at the angle of  $360^\circ$  (fig. 79/5). In order for the vault to be hollow, it was divided into parts using the option **Mini Menu/Explode**, and then the bases of the cylinder were deleted. If necessary, the cylindrical shell can be given thickness using the option **Solid/Offset**, whereby it is necessary first to join all three elements into one whole using the option **Join**.

## EXERCISE 6 – Rhinoceros: Model the given object using cone

### Procedure for constructing conical fold structure from figure 77:

1. **File/Properties/Units/Units and tolerances/Model units/Meters** – adjusting the units in which it is drawn;
2. Using the option **Solid/Cone**, **Radius 5m** in **Front** view, draw cone basis, then give height in **Right** view – **End of Cone 20m**;
3. Using the option **Transform/Rotate** in **Right** view, rotate cone for angle  $-25^{\circ}$  in relation to center of basis;
4. In **Top** view using the option **Transform/Array/Polar Center of polar** (*top cone*) **Number of items 12**, **Angle  $360^{\circ}$**  radial cone copy;
5. Include **Osnap/Int**;
6. Using the option **Surface/Plane/Cutting Plane** in **Front** view, set horizontal plane through upper section of cone basis (*include **Osnap/Intersection***);
7. Using the option **Trim** reject all that is below horizontal plane;
8. Using the option **Cut** or **Delete** on keyboard delete horizontal plane;
9. The inside of the object can be arranged by placing vertical planes:
  - a) **Surface/Plane/Cutting Plane** in **Top** view, set plane through one of intersection of cones;
  - b) **Transform/Array/Polar, Center of polar** (*center of an object*), **Number of items 7**, **Angle  $180^{\circ}$**  copy vertical plane radially;
  - c) Using the option **Split** cut cones with vertical planes;
  - d) Using the option **Cut** or **Delete** on keyboard, delete all vertical planes and parts of cone that are inside of the object;
10. Using the option **Explode**, basis is separated from the cone wrapper. It is possible to delete bases and to set curved surfaces:
  - a) Select whole object, **Explode**;
  - b) Delete all cone bases;
  - c) In **Perspective** or **Top** view, draw a line that connects two opposite common points of bases of adjacent cones, so that middle point of the line is on the same vertical as tops of cones. Include **Osnap/Mid** in **Perspective** view, draw circle whose center is middle point of that line, and radius ends in intersection point of cone bases (see the right sketch);
  - d) Using **Surface/Extrude Curve/Straight** option in **Perspective** view, make cylinder out of circle height up to top of cones;
  - e) Using **Trim** option in **Perspective** view, select all cones and then delete part of cylinder that is above conical shells;
11. Using the option **Solid/Offset** in **Top** view select all shells of cone and give them thickness of 0.2m;



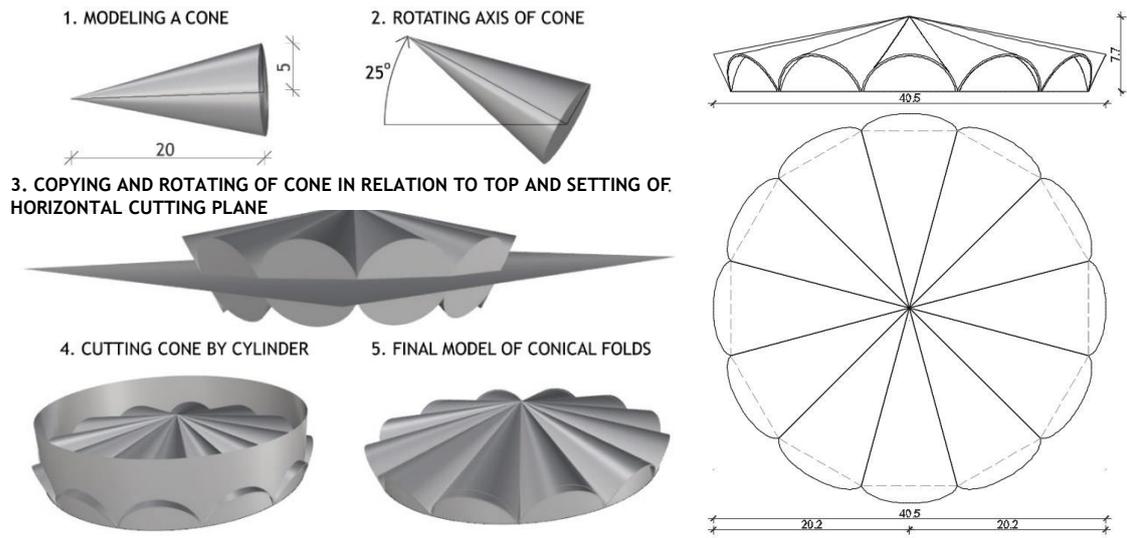


Figure 77 – Procedure for modelling conical folds (left) and top and front view (right)

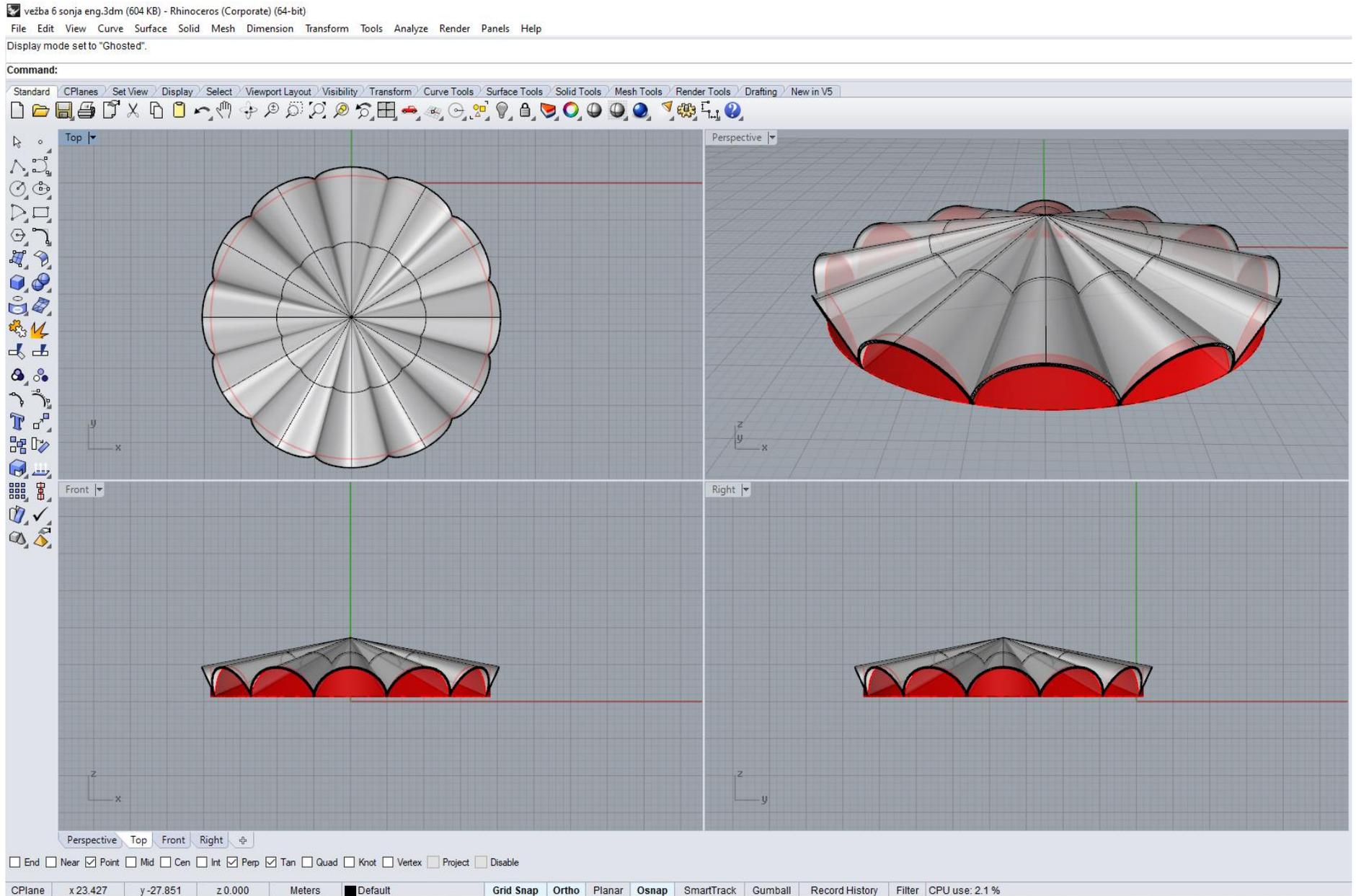


Figure 78 – Example Exercise 6

## HOMEWORK 6: Model the given object using cylinder

Procedure for constructing cylindrical vault over triangular basis from figure 79:

1. In **Top** view draw equilateral triangle side 20m using the option **Curve/Polygon/By Edge** and find its orthocenter using the option **Curve from object/Intersection (intersection of heights)**;
2. Using the option **Solid/Cylinder** in **Front** view, draw cylinder basis over one of the triangle sides, and then in **Top** view, define length of the surface to triangle orthocenter;
3. Using the option **Surface/Plane/Cutting Plane** in **Front** view, set horizontal plane through center of cylinder basis;
4. Using the option **Trim**, reject the part of cylinder which is below the horizontal plane;
5. Delete the horizontal plane;
6. In **Top** view, set two vertical planes through appropriate heights of triangle;
7. Using the option **Trim** reject parts of cylinder which are outside of triangle;
8. Using the option **Transform/Array/Polar, Center of polar (triangle orthocenter), Number of items 3, Angle 360°** copy and rotate slice of cylinder in **Top** view;

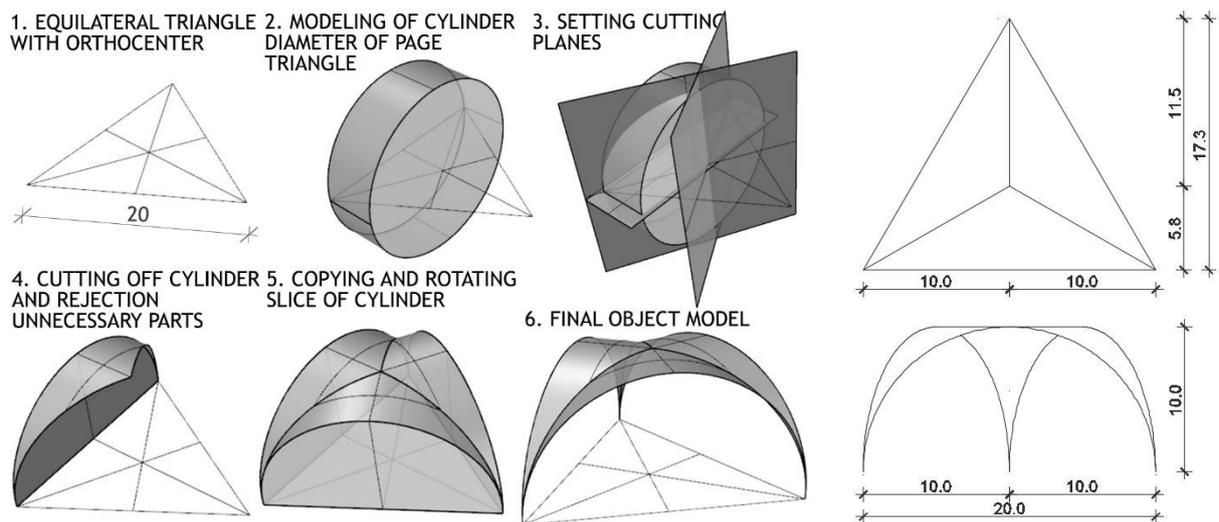


Figure 79 – Procedure for modelling cylindrical vault (left) and top and front view (right)

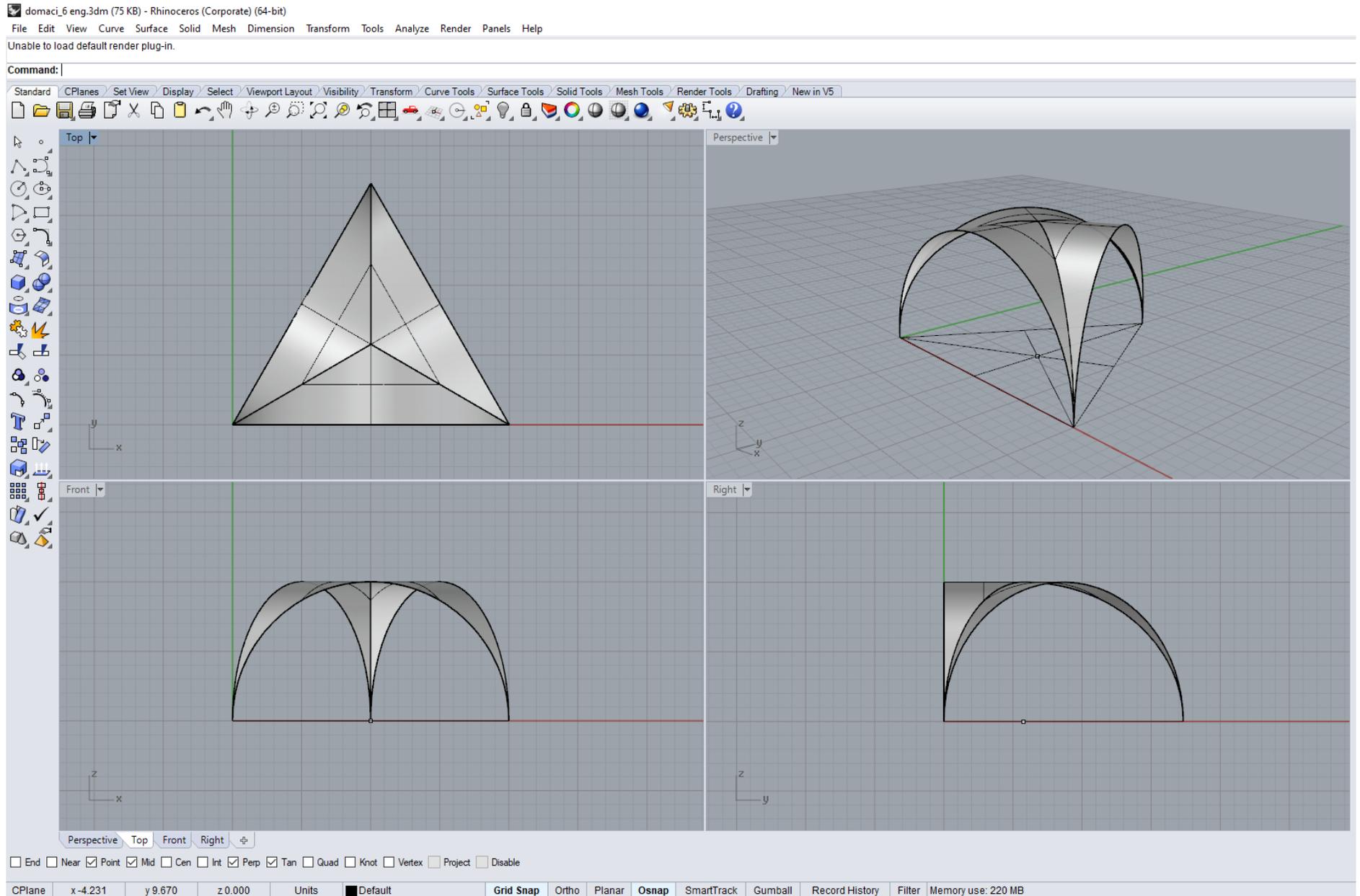


Figure 80 – Example Homework 6

Figures 81 and 82 show representative examples of the use of cone and cylinder fold in architecture.



Figure 81 – Conical folds, Chemical Sciences Auditorium, Mexico, arch. F. Candela, year 1952



Figure 82 – Cylindrical folds, Church of Saint Francis of Assisi, Mexico, arch. O. Niemeyer, year 1943

## UNIT 7 – HELICAL SURFACES WITH LINE GENERATRIX: HELICOIDS

### HELIX – SPATIAL CURVE

Since the helix, which is kind of a spatial curve, is commonly used in technical practice, in this part helix will first be considered as one curve, **directrix** (guideline) of helical surface.

There are three types of the helix:

1. **cylindrical** – when one point moves translationally along a straight line, with simultaneous rotation around a parallel line (along the cylinder wrapper, fig. 83)
2. **conical** – when one point moves translationally along a straight line, with simultaneous rotation around the line with which it intersects (along the cone wrapper, fig. 84)
3. **spherical** – when one point moves translationally, with simultaneous rotation, so that this path intersects the meridians at the same angle (along the sphere wrapper, fig. 85)

Cylindrical helix has greatest application in architectonic practice.

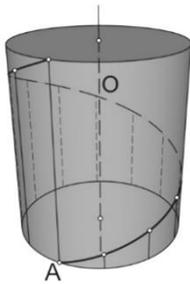


Figure 83 – Cylindrical helix



Figure 84 – Conical helix

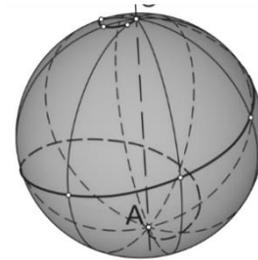


Figure 85 – Spherical helix

### HELICOIDS

In general, there are three **directrices** (guidelines) of non-developable surface and those are three curves, but there may also be some other combinations.

Helicoids are helical surfaces whose **generatrices**  $g$  are straight lines and **directrices** (guidelines)  $d$  are cylindrical helix. Non-developable, helical surfaces belong to the group of ruled surfaces which has only one generatrix system. Helicoids can be: closed – if the generatrices intersect axis of the surface; and open – if generatrices do not intersect axis of the surface.

**Directrices** of these surfaces can be two helical curves and one infinitely distant line or a helical curve and one line in finiteness and infinitely distant curve. According to how generatrix stands in relation to the axis of surfaces, helicoids can be:

**normal** helicoid – generatrices  $g$  are perpendicular to the axis  $a$  (fig. 86),

**oblique** helicoid – generatrices  $g$  are inclined to the axis  $a$  (fig. 87).

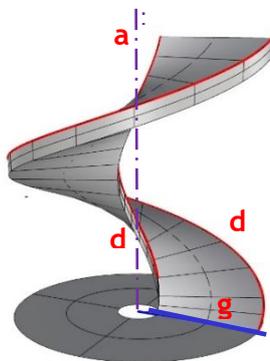


Figure 86 – Normal helicoid

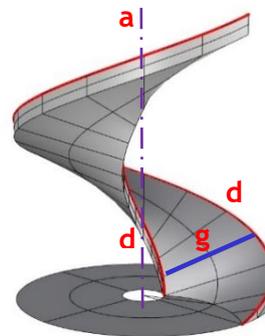


Figure 87 – Oblique helicoid

# MODELLING OBLIQUE AND NORMAL HELICAL STAIRCASE IN THE COMPUTER SOFTWARE RHINOCEROS

## EXERCISE 7

EXERCISE 7 showed how to construct helical staircase with railing (fig. 88), in 19 steps, with given dimensions. That is why it is necessary to change measurement units from mm to m using the option **File/Properties/Units**. The panel of the staircase is an oblique helicoid, whose directrices are two cylindrical helices. Firstly, it is necessary to draw two concentric circles, the first projections of the helices, with radius  $r=1.2\text{m}$  and  $r=0.2\text{m}$ , respectively, using the option **Curve/Circle** in **Top** view. The smaller circle is at the same time the projection of the pillar, which supported the console staircase. The option **Curve/Helix** is used to draw two helices with different radius and the starting height (fig. 88/1). The generatrix of the helicoid is an oblique line, generated by the option **Curve/Line**, by merging the starting points of the helices. Using the option **Surface/Sweep 2 Rail**, an oblique helicoid was modelled (stair panel, fig. 88/2). In order to model one stair, it is necessary to divide spherical surface (between two concentric circles), generated by the **Surface/Loft** option, into 12 parts using line through center of circles which is radially copied by **Transform/Array/Polar** command (fig. 88/3). Then, 11 parts of the spherical surface need to be removed and the solid of the stair formed with height  $h=17.5\text{cm}$  using the command **Extrude Surfaces/Straight** in **Perspective** view. The same option is used to form the solid of the pillar, height  $h=2.8\text{m}$ . The option **Transform/Array/Along Curve/Method/Number of items 17/Orientation Roadlike**, makes it easy to copy all steps along the bigger helix, whereby the last step is removed (fig. 88/4).

**The pillar and the stairs, that had been modelled, need to be copied to use it for the homework.**

The stair panel is assigned the height  $h=0.175\text{m}$ , under the steps, using the command **Solid/Extrude Surfaces/Straight**, best in **Perspective** view. To model the hand railing, bigger helix is copied and elevated to the appropriate height,  $h=1.375\text{m}$ . It is necessary to create a circle with radius  $r=2.5\text{cm}$ , perpendicular to the helix direction, using the option **Curve/Circle/Around Curve**, in **Perspective** view. Using the option **Surface/Sweep 1 Rail/Style=Roadlike** in **Top** view, a surface is created, which represents the thickness of the railing, and the empty spaces are closed off using the option **Cap** (fig. 88/5).

## HOMEWORK 7

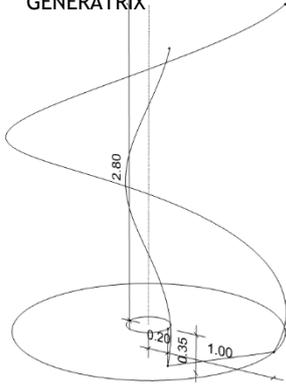
HOMEWORK 7 models helical staircase with the same dimensions as in Exercise 7, but with a panel which is a normal helicoid (fig. 90). That is why everything that had been done in exercises is copied, up to step 13. The only thing that changes is the panel that is formed from the same helices as in the exercises, with equal starting height (fig. 90/1). In this case, the generatrix is a line which stands perpendicular to the helix axis, which is why the surface modelled using the option **Surface/Sweep 2 Rail**, is a normal helicoid (fig. 90/2). This surface also needs to be given height  $h=0.175\text{m}$ , under the steps, using the command **Solid/Extrude Surfaces/Straight**, best in **Perspective** view. This staircase also needs a railing that can be modelled in the same way as during the exercises, or, previously modelled railing can be copied and moved (fig. 90/3).

## EXERCISE 7 – Rhinoceros: Model the given object using an oblique helicoid

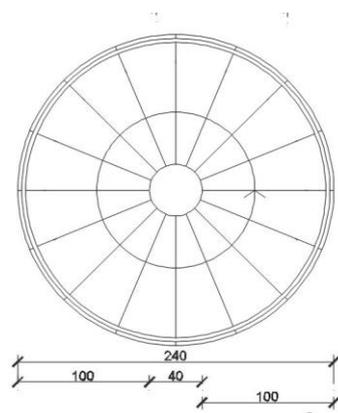
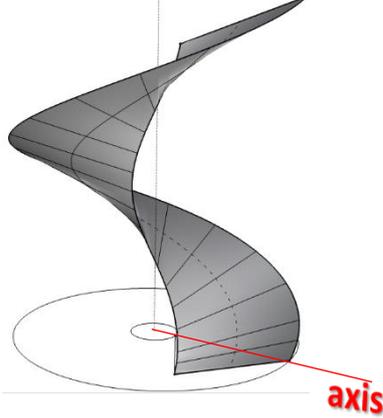
### Procedure for constructing helical staircase – oblique helicoid from figure 88:

1. Draw larger circle  $r=1.2\text{m}$  and smaller one  $r=0.2\text{m}$  with same center, using the **Curve/Circle** option in **Top** view (*concentric circles*);
2. Draw one helix with radius of a larger circle and height of  $2.8\text{m}$  and the other with radius of a smaller circle and the same height (*center of helix is the center of pillar in basis*), by **Curve/Helix/Turns = 1** option, in **Front** view;
3. Put the smaller helix down vertically to height of two stairs, total of  $35\text{cm}$  ( $h_{\text{step}}=17.5\text{cm}$ ), using the option **Transform/Move**, in **Front** view;
4. In **Perspective** view, draw a generatrix at an angle, connecting lower initial points of both helices using the option **Curve/Line**;
5. Using the option **Surface/Sweep 2 Rail** in **Perspective** view, form a helicoid, (*select first helix, then second helix and then generatrix at an angle*);
6. In the same view, using the option **Surface/Loft** make a surface between two concentric circles;
7. Using the option **Mini Menu/Trim** in **Perspective** view remove part of helicoid which is obtained under horizontal circular surface;
8. Using the option **Curve/Line** in **Perspective** view, draw radius of larger circle containing the beginning of the larger helix (*include Osnap/End and Perp*);
9. Include **Osnap/Cen** and then using the option **Transform/Array/Polar/Number of items 2/Angle  $22.50^\circ$**  in **Perspective** view, copy and rotate radius of a larger circle relative to its center, so that it is the tread of one stair (*copy and move, simultaneously, horizontal circular surface*);
10. Using the option **Mini Menu/Trim** in **Perspective** view, cut-off horizontal circular surface by radius are drawn in same plane and reject most of surfaces except for the tread of one stair;
11. Using the option **Solid/Extrude Surfaces/Straight** in **Perspective** view, make stair of horizontal circular slice (*tread*) of height  $17.5\text{cm}$ ;
12. Get a pillar in the middle by raising smaller circle using the option **Solid/Extrude Planar Curve** at the height of  $2.8\text{m}$  in **Perspective** view;
13. Using the option **Transform/Array/Along Curve/Method/Number of items 17/Orientation Roadlike** choose **Top** view, copy and rotate stairs on a larger helix (*delete last stair at the end as excess*);
14. Using the option **Solid/Extrude Surfaces/Straight** in **Perspective** view, by selecting oblique helicoid give it thickness of  $17.5\text{cm}$  downwards to obtain panel below stairs;
15. Use the option **Transform/Copy** to copy larger helix at height of railing  $h=1.375\text{m}$ , in **Perspective** view;
16. Create circle radius  $r=2.5\text{cm}$  normal to helix direction with **Curve/Circle/Around Curve** option, in **Perspective** view;
17. Using the option **Surface/Sweep1 Rail/Style=Roadlike** in **Top** view give the thickness to railing;
18. Using the **Cap** option close gaps on the railing;
19. If desired, draw a vertical part of the railing;

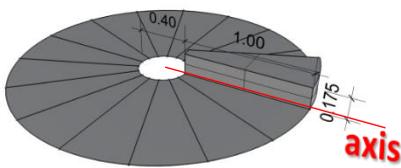
1. DRAWING CIRCLES AND BOTH HELICES WITH HELICOID GENERATRIX



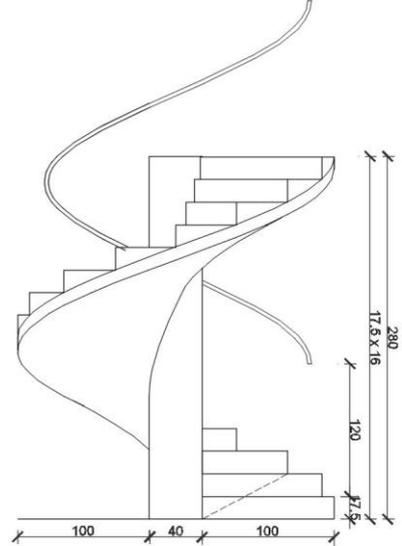
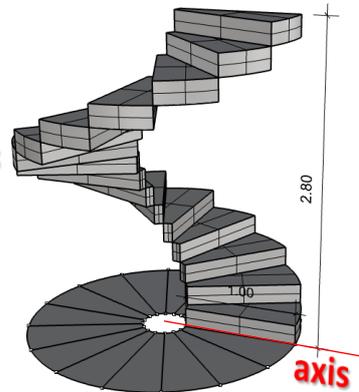
2. FORMATING SURFACE OF TWO HELICES AND OBLIQUE GENERATRIX



3. MODELING ONE STEP, FOR HELICAL STAIRCASE APPROPRIATE HEIGHT AND WIDTH



4. LIFTING ALL STEPS TO ENTIRE HEIGHT OF HELICOID



5. FINAL MODEL OF HELICAL STAIRCASES

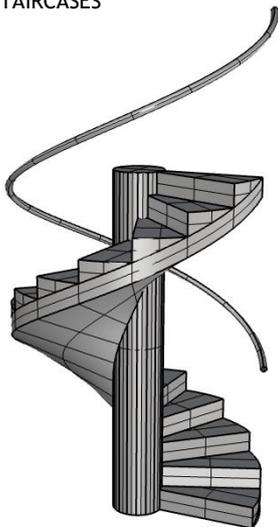


Figure 88 – Procedure for modelling helical staircases (left) and top and front view (right)

Select a display mode ( Viewport=Active Mode=Shaded ): \_Mode=\_Shaded

Display mode set to "Shaded".

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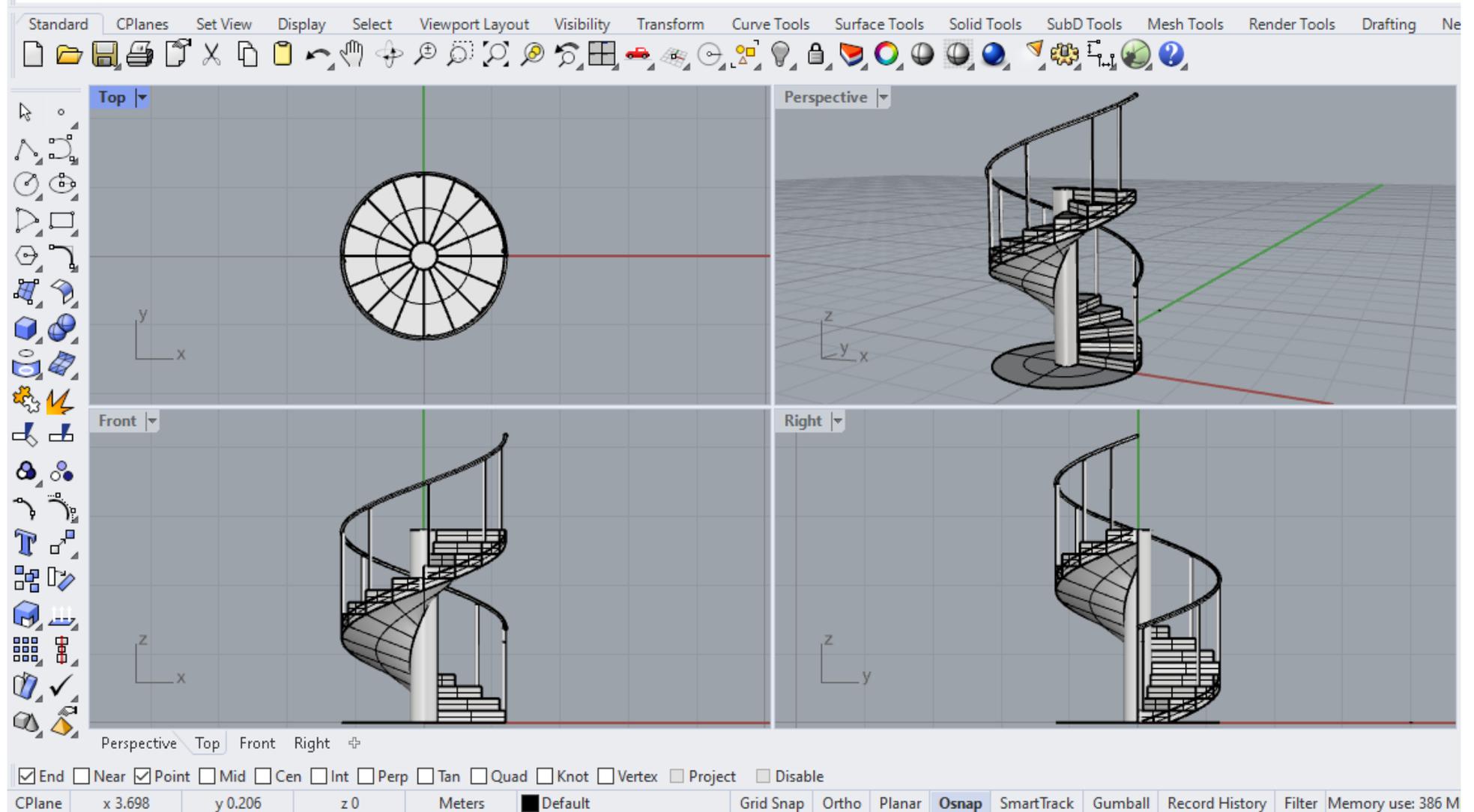


Figure 89 – Example Exercise 7

## HOMEWORK 7: Model the given object using normal helicoid

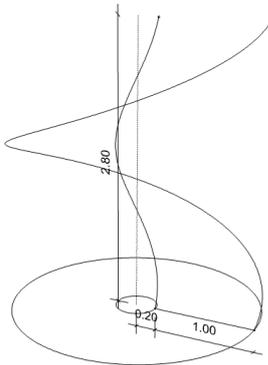
### Procedure for constructing helical staircase – normal helicoid from figure 90:

*Note: Use stairs with the pillar from the exercise in order to model stairs for the homework, because they do not differ; the only difference is in the stair panel;*

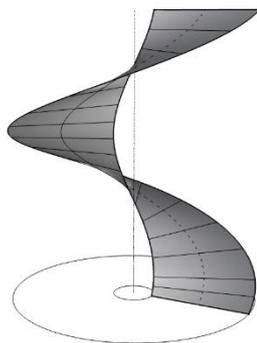
1. Draw one helix with the radius of bigger circle  $r=1.2\text{m}$  and height  $2.8\text{m}$  and another one with radius of smaller circle  $r=0.2\text{m}$  the same height (*center of the helix is the center of the column in the basis*), using the option **Curve/Helix/Turn=1** in **Front view**;
2. Draw a generatrix of the normal helicoid using the option **Line** in **Front view** so that the first point is the beginning point of the smaller, and the second point the beginning point of the bigger helix;
3. Form the helicoid using the option **Surface/Sweep2 Rail/Select Roadlike Top** in **Perspective view**;
4. Using the option **Solid/Extrude Surfaces/Straight** in **Perspective view**, selecting the normal helicoid, give it the thickness of  $17.5\text{ cm}$  downwards so that we obtain a helical panel under the steps;

*Note: The rest of the homework is done using the same procedure as during the exercises;*

1. DRAWING CIRCLES AND BOTH HELICES WITH HELICOID GENERATRIX



2. FORMATING SURFACE OF TWO HELICES AND HORIZONTAL GENERATRIX



3. FINAL MODEL OF HELICAL STAIRCASE

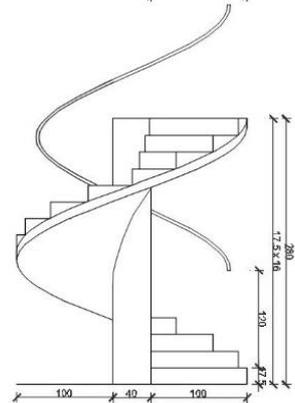
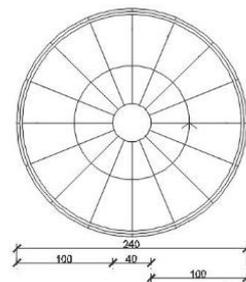
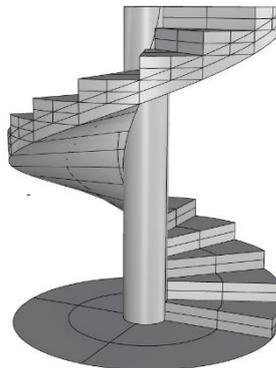


Figure 90 – Procedure of modelling helical staircase (left) and top and front view (right)

2 curves added to selection.

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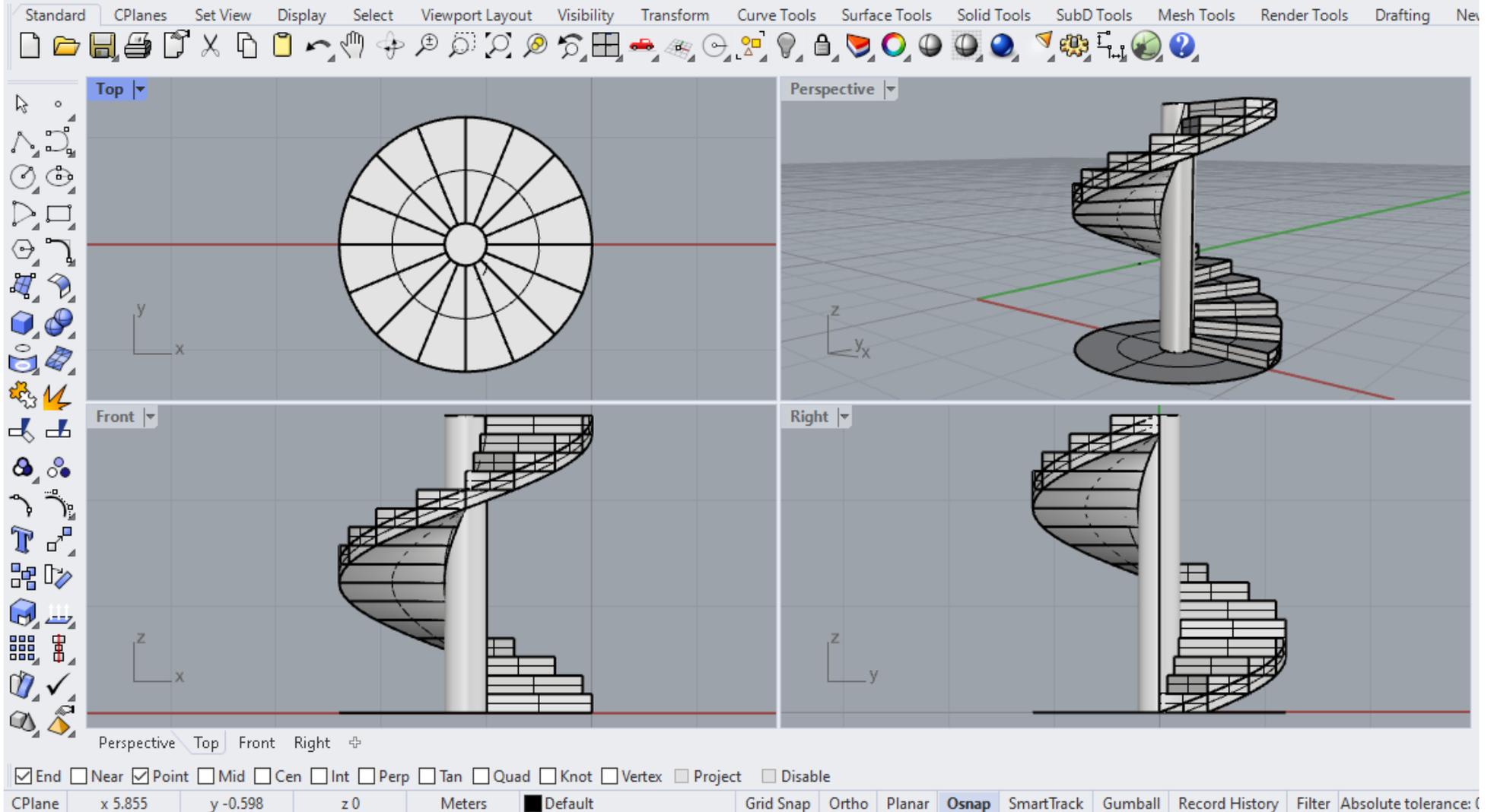


Figure 91 – Example Homework 7

Figures 92, 93 and 94 show representative examples of using oblique and normal helicoid in architecture.



Figure 92 – Oblique helical staircase, Garvan Institute, Australia



Figure 93 – Oblique helical staircase



Figure 94 – Normal helical staircase, Town hall, London, England, arch. N. Foster, year 2002

## UNIT 8 – RULED SURFACES WITH ONE GENERATRIX SYSTEM: CONOIDS AND CYLINDROIDS

### CONOIDS

Conoid (fig. 95) is a ruled, non-developable surface with one generatrix system which is generated with one line in finiteness  $d_1$ , one spatial or planar curve  $d_2$ , and one infinitely distant line  $b^\infty$  specified by directrix plane  $\Delta$ , all three as **directrices** (guidelines). Directrix plane  $\Delta$  can be horizontal or vertical.

**Generatrices** of conoid are parallel to directrix plane  $\Delta$  and are cutting guidelines  $d_1$  and  $d_2$ . In the horizontal projection plane **generatrices** are parallel to trace  $\Delta_1$ . Spatial projection of **generatrices** that are parallel to directrix plane  $\Delta$  are mutually skew.

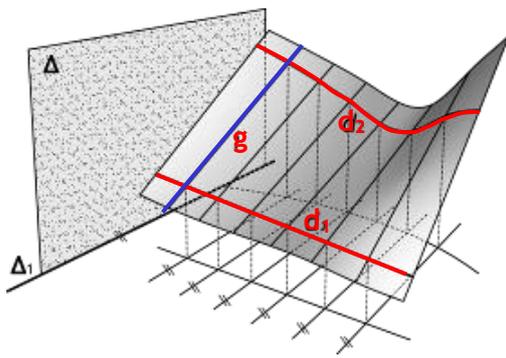


Figure 95 – Conoid

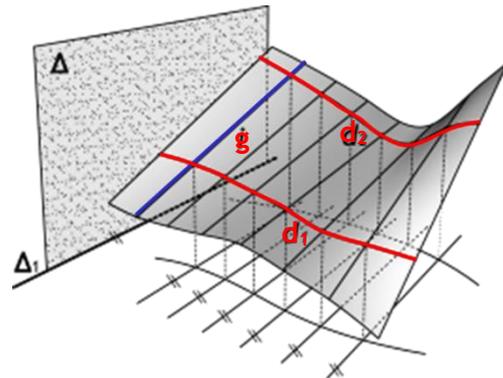


Figure 96 – Cylindroid

### CYLINDROIDS

Cylindroid (fig. 96) is a ruled, non-developable surface with one generatrix system, which is generated with two spatial or planar curves  $d_1$  and  $d_2$  and one infinitely distant line  $b^\infty$ , specified by directrix plane  $\Delta$ , all three as **directrices** (guidelines). Directrix plane  $\Delta$  can be horizontal or vertical.

**Generatrices** of cylindroid are lines, parallel to directrix plane  $\Delta$  and they cut directrices  $d_1$  and  $d_2$ . In the horizontal projection plane, **generatrices** are parallel to track  $\Delta_1$ . Spatial projection of generatrices, that are parallel to directrix plane  $\Delta$  are not mutually parallel.

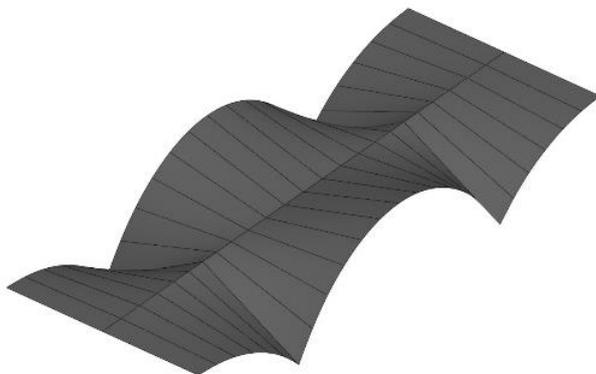


Figure 97 – Example of Conoid

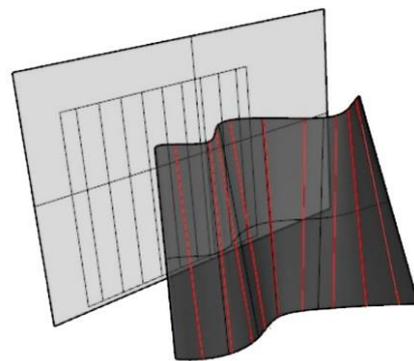


Figure 98 – Example of Cylindroid

## MODELLING CONOIDAL CANOPY AND CYLINDROID IN THE COMPUTER SOFTWARE RHINOCEROS

### EXERCISE 8

EXERCISE 8 showed how to construct conoidal canopy (fig. 99) in 10 steps, with given dimensions. That is why it is necessary to change measurement units from mm to m using the option **File/Properties/Units**. The canopy consists of conoidal panel and supporting pillars. The basic conoid is given with two directrices (guidelines), one semi-circle in vertical and one straight line in horizontal plane, over the opposite sides of a square, side  $a=6\text{m}$ , generated using the option **Curve/Rectangle/Corner to corner** in **Top** view. The semicircle is drawn in vertical plane, in **Front** view, over one side of the square, using the option **Curve/Circle** and then the lower half is rejected by using the option **Trim**. Since the square is drawn as a polyline, it is possible to separate its sides using the option **Explode**, or draw a new line opposite of the plane in which is the semicircle. The basic conoid is modelled using the option **Surface/Loft**, whereby the semicircle and the straight line above the opposite sides of the square are chosen for merging (fig. 99/1). The basic surface is cut across the diagonal of the square and its lower part was removed in **Top** view (fig. 99/2). By mirroring the clipping, using the option **Transform/Mirror**, in relation to the diagonal of the square, one quarter of the canopy is generated (fig. 99/3). The thickness of the conoid towards the inside,  $h=0.2\text{m}$ , is generated by using the option **Solid/Offset**, in **Perspective** view. Using the option **Solid/Box/Corner to Corner**  $0.2\times 0.2\text{m}$ , **Height**  $4\text{m}$ , one pillar supporting the canopy was modelled, and then it was copied 3 times, in the corners of the basic square. Since the conoidal panel and pillars are separate elements, it is necessary to merge them into one whole using the option **Join**, best in **Perspective** view. Then by using the option **Transform/Array/Polar**, one quarter was radially copied in order to obtain the whole canopy (fig. 99/4).

### HOMEWORK 8

HOMEWORK 8 is to model a cylindroid (fig. 101), in 7 steps, with given dimensions. That is why it is necessary to change measurement units from mm to m using the option **File/Properties/Unit**. Directrices (guidelines) of the cylindroid are two semicircles, of the same radius  $r=5\text{m}$ , that are in vertical, mutually perpendicular planes (fig. 101/1). The first semicircle is drawn in **Front** view, using the option **Curve/Circle/Line**, and the option **Mini/Menu/Trim** is used to discard the lower half. Then the semicircle was copied, moved for  $15\text{m}$  and rotated in relation to the center, using the option **Transform/Rotate** at an angle of  $90^\circ$ , to obtain the second guideline. Cylindroid was modelled using the option **Surface/Loft** (fig. 101/2). Horizontal intersecting planes were set in **Front** view, at height of  $1\text{m}$  (fig. 101/3), and then the option **Curve from Objects/Intersection** has shown that the intersecting lines, generatrices of the cylindroid, are in horizontal planes, because the directrix plane is horizontal (fig. 101/4).

## EXERCISE 8 – Rhinoceros: Model the given object using conoid

### Procedure for constructing conoidal canopy from figure 99:

1. In **Top** view, draw square of 6x6m with **Curve/Rectangle/Corner** to corner option;
2. In **Front** view, draw a semicircle above one of the square sides using the options **Circle, Line, Trim**;
3. In **Perspective** view, using the option **Surface/Loft**, connect the semicircle with the opposite side of square (if square is polyline, lines are separated with **Explode** option);
4. Using the option **Surface/Plane/Plane Cutting** in **Top** view, set vertical intersection plane along the diagonal of square;
5. Use **Trim** option to remove lower part of conoid;
6. Using the option **Cut** or **Delete** on keyboard delete vertical plane;
7. Using the option **Transform/Mirror** in **Perspective** view, mirror part of conoid in relation to the diagonal of square;
8. In **Perspective** view, give thickness of conoid to inside (down) using the option **Solid/Offset**, 0.2m, take care of direction of vector (direction of vector changes by left clicking on surface);
9. In **Perspective** view, make a pillar using the option **Solid/Box/Corner** to **Corner** 0.2x0.2m, **Height** 4m and using the option **Transform/Copy** copy pillars in corners of the square in basis as supports for conoids;
10. Using the option **Transform/Array/Polar**, **Number of items** 4, **angle** 360° rotate and copy conoids with pillars;

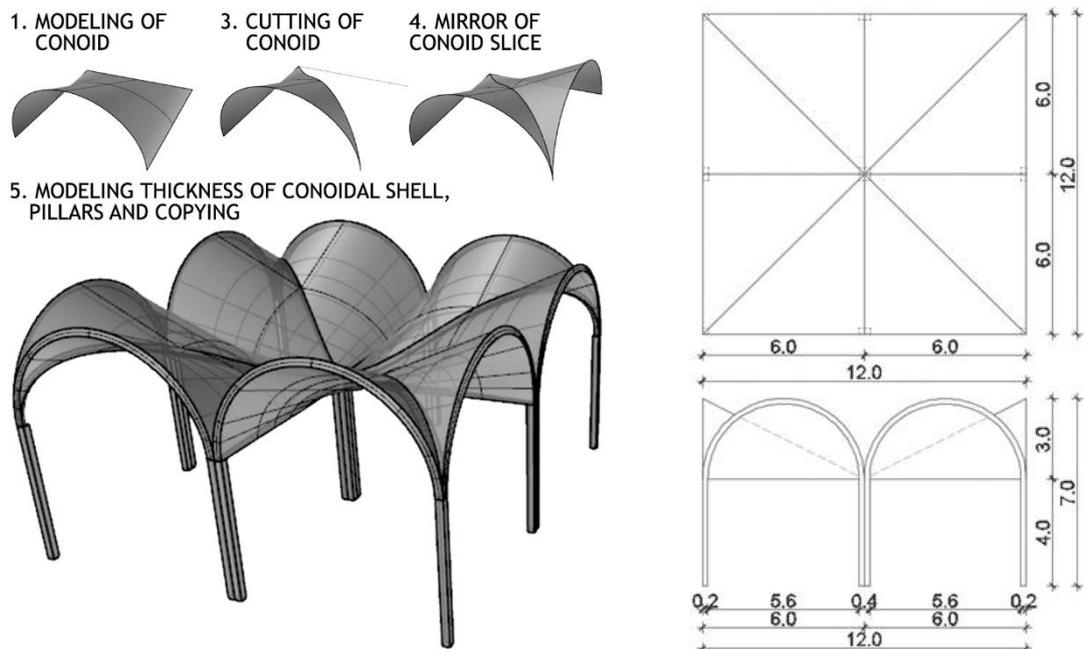


Figure 99 – Procedure for modelling conoidal canopy (left) and top and front view (right)

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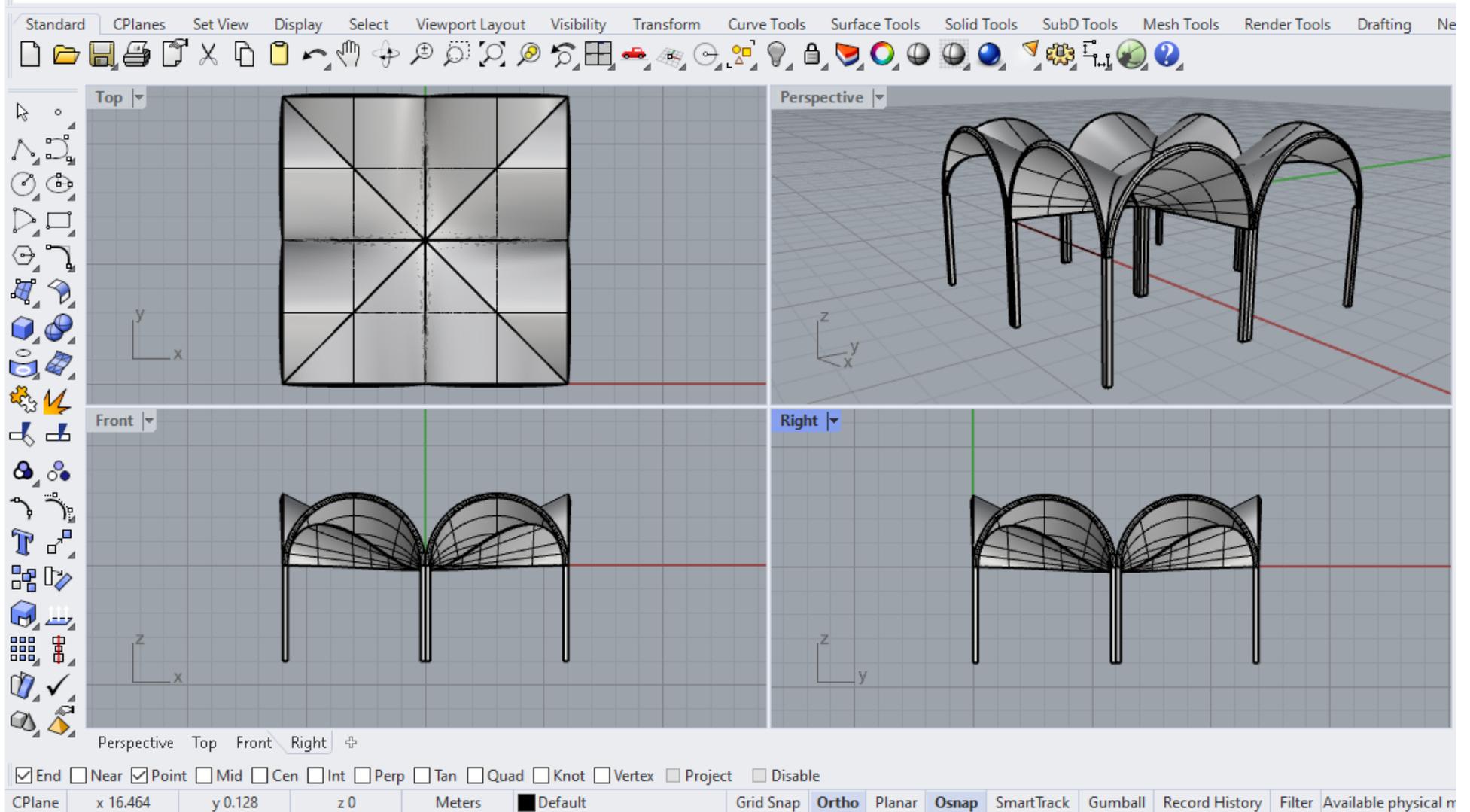


Figure 100 – Example Exercise 8

## HOMEWORK 8: Model the given object using cylindroid

### Procedure for constructing cylindroid from figure 101:

1. In **Front view**, draw a semicircle with radius 5m (**Circle, Line, Trim**);
2. In **Top view**, copy the semicircle at the distance of 15m;
3. Using the option **Transform/Rotate**,  $90^\circ$  rotate the distant semicircle at  $90^\circ$  in relation to its center;
4. In **Perspective view**, using the option **Surface/Loft**, connect the semicircles to model a cylindroid;
5. In **Front view** set the horizontal cutting planes at 1m using the option **Surface/Plane/Cutting Plane**;
6. Using the option **Curve/Curve from Objects/Intersection** find the cross sectional planes with cylindroid;
7. Delete horizontal planes using the option **Delete** on the keyboard.

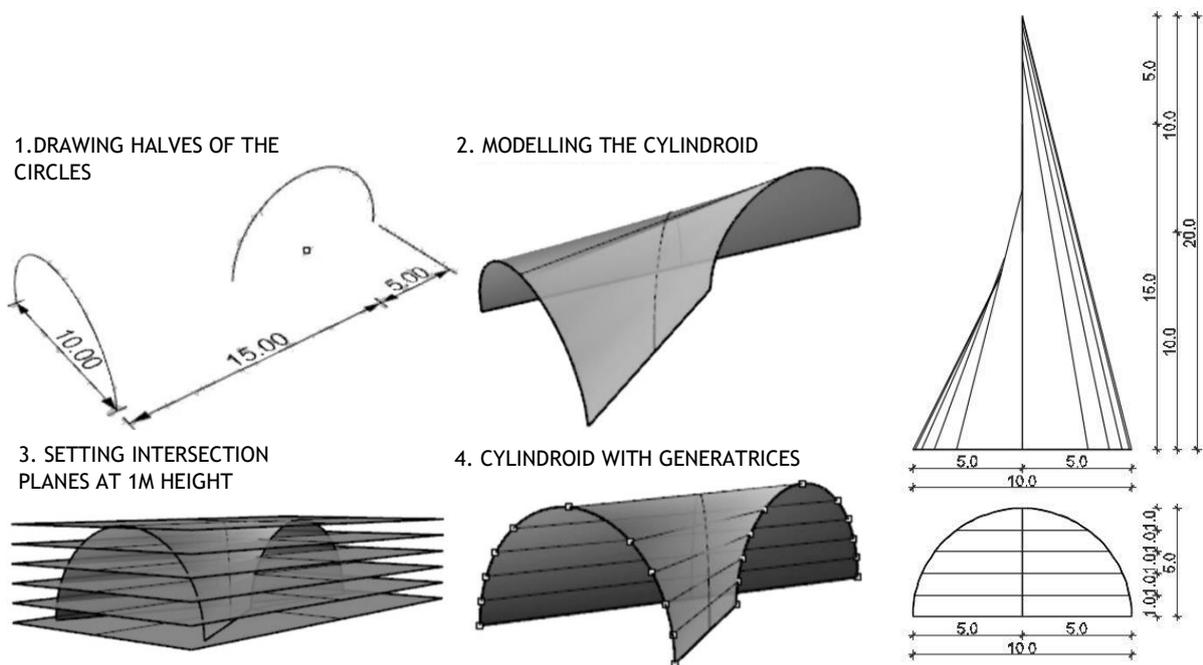


Figure 101 – Procedure for modelling cylindroid (left) and top and front view (right)

Command: |

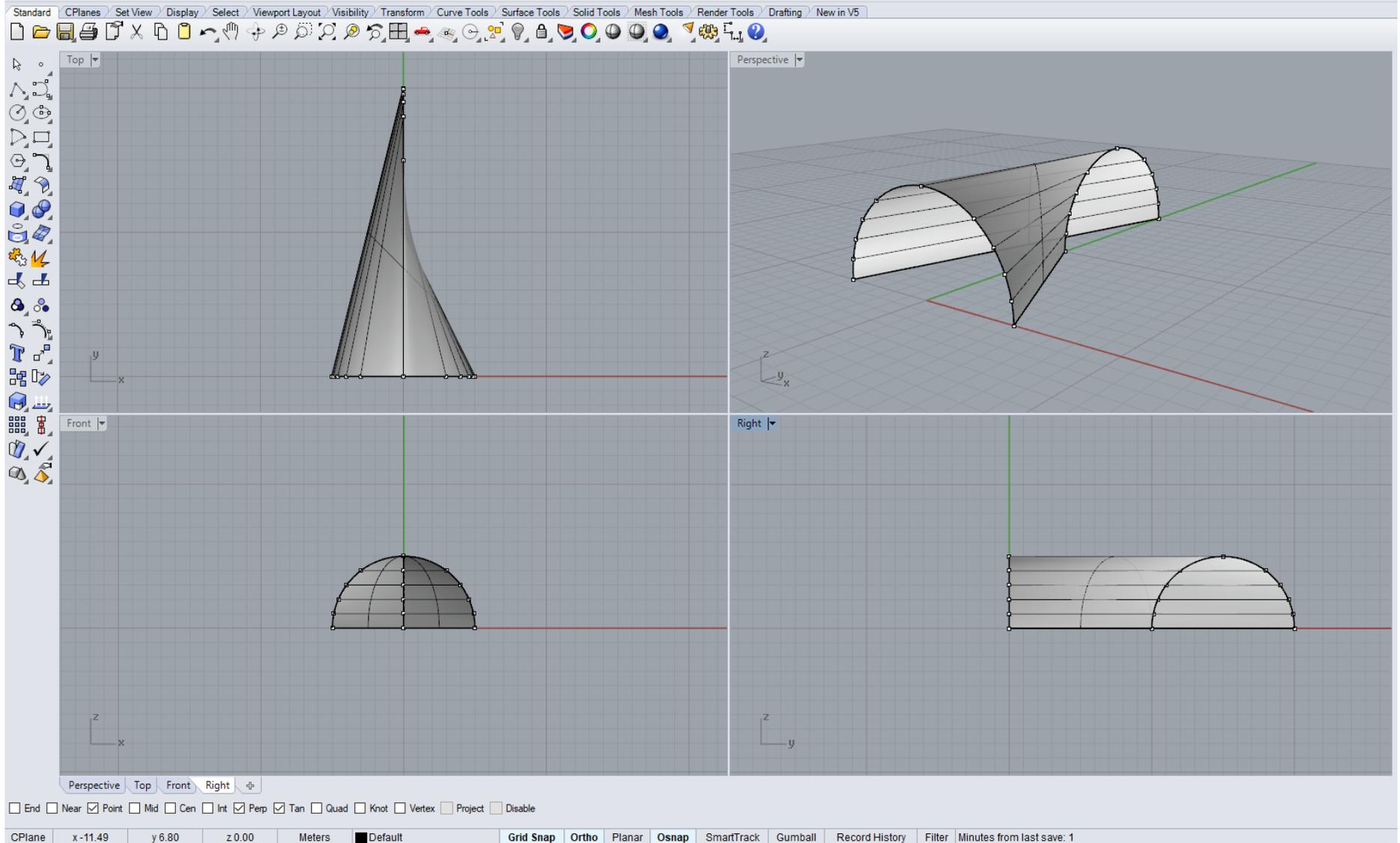


Figure 102 – Example Homework 8

Figures 103 and 104 show representative examples of the use of conoids and cylindroids in architecture.



Figure 103 – Conoids, Milwaukee Art Museum, Lake Michigan, USA, arch. S. Calatrava, year 2001



Figure 104 – Cylindroids, Disney Concert Hall, Los Angeles, USA, arch. F.O.Geri, year 2003

# UNIT 9 – RULED SURFACES WITH TWO GENERATRIX SYSTEMS: HYPERBOLIC PARABOLOIDS

## SPECIAL ROTATIONAL SURFACE: SPHERE (BALL)

### HYPERBOLIC PARABOLOID

Hyperbolic paraboloid (HP) is one of the non-developable, ruled surfaces of II degree, with two generatrix systems, because all three **directrices** (guidelines) are lines, two in finiteness and one infinitely distant (fig. 105 and 106). Since HPs can be generated by translational movement of one curve above another, they also belong to the translational surfaces.

Hyperbolic paraboloid is generated in two ways:

- a) By spatial quadrangle, three **directrices**, two skew lines **a** and **b** in finiteness, and one infinitely distant line **b $\infty$** , specified by directrix plane  $\Delta$  (fig. 105).
- b) By translating of first parabola **p<sub>1</sub>** along the second parabola **p<sub>2</sub>**, which are lying down in perpendicular planes and touch each other at vertices points (fig. 106).

The **first generatrices system** **g<sub>1</sub>** cuts the skew lines **a** and **b**, whereby generatrices are parallel to the directrix plane  $\Delta$ , which is determined with **c** and **d** lines. **Second generatrices system** **g<sub>2</sub>** is obtained by taking any two of the first generatrices system for directrices in finiteness, in this case **c** and **d**, and infinitely distant line specified directrix plane is parallel to the skew lines **a** and **b** (fig. 105).

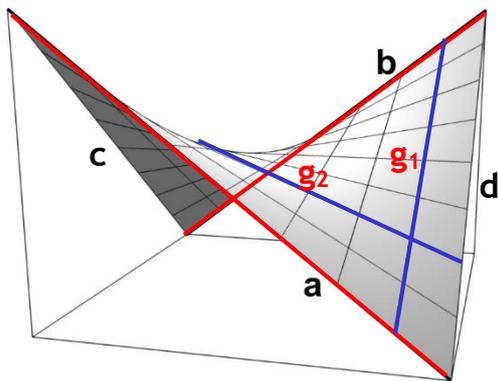


Figure 105 – Hyperbolic paraboloid, by spatial quadrangle

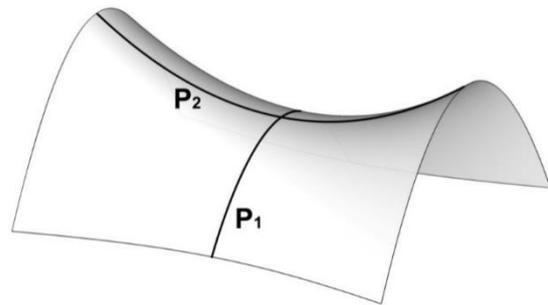


Figure 106 – Hyperbolic paraboloid, by movement of two parabolas

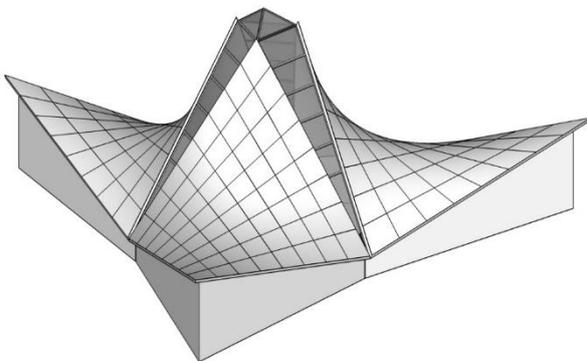


Figure 107 – HP, Chapel of San Vicente de Paul, Mexico, arch. F. Candela, year 1960

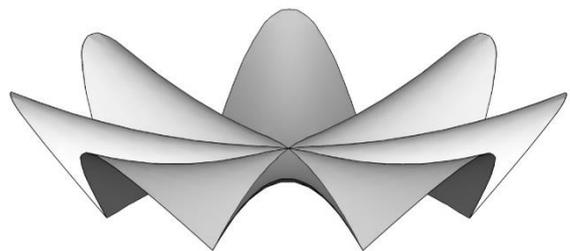
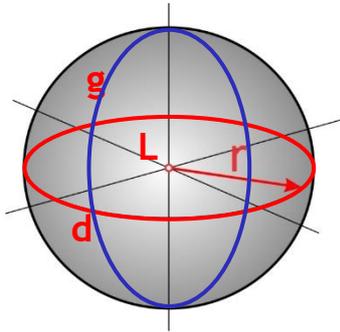


Figure 108 – HP, Los Manantiales restaurant, Mexico, arch. F. Candela, year 1958

## SPHERE (BALL)

If a certain curve **g** **generatrix** rotates to the second curve **d** **directrix** (guideline), the generated surface is rotational. Among all general surfaces, only the general surface of II degree (quadrics) will be discussed, which has been used a lot in architecture.



Sphere is generated by rotation of **generatrix**, circle **g**, around the **directrix** of circle **d** through the center **L** of the sphere (fig. 109). Because of that, the sphere belongs to special rotational surfaces. The sphere is geometrical place of points which have the same distance from constant point, the center **L** of the sphere, and that distance is the radius **r**. Orthogonal projections of the sphere, the center **L** and radius **r** are circles of radius **r**. Axonometry projection of sphere is a circle, center **L** and radius **r** (fig. 109).

Figure 109 – Sphere

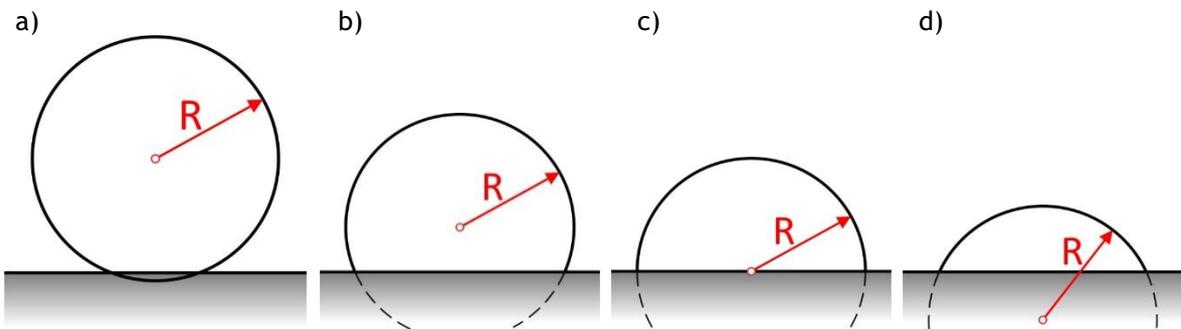


Figure 110 – Various parts of the sphere in architectural form a) almost whole (90%); b) as part of; c) dome (hemisphere); d) calotte (slice);



Figure 111 – Sphere, International Center, Japan, arch. M. Murata and J. Cuboi, year 1959

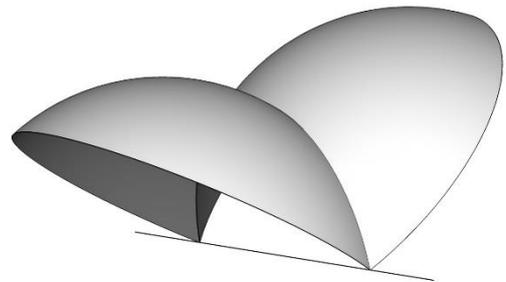


Figure 112 – Example of sphere

## MODELLING HYPERBOLIC PARABOLOID'S OBJECT AND SPHERICAL OBJECT IN THE COMPUTER SOFTWARE RHINOCEROS

### EXERCISE 9

EXERCISE 9 showed how to construct an object obtained by merging 3 hyperbolic paraboloids (HP), in 16 steps, with given dimensions. That is why it is necessary to change measurement units, from mm to cm, using the option **File/Properties/Units**. That is Chapel St. Vicente de Paul, Mexico City, architect F. Candela (fig. 107 and 113). Firstly, in **Top** view, a deltoid **ABCD** was constructed using the option **Curve/Polyline**, which is the basis of the spatial quadrangle of an HP, using the sides **AB**, **AD** and the diagonal **AC** (fig. 113/1). Since it was necessary to find a spatial quadrangle by which the HP was given, the vertices of the deltoid were marked using the option **Mini Menu/Text + Point Object/Single Point**. The walls of the object are indented in relation to the sides of the spatial quadrangle **BC** and **CD**. That is why in **Top** view, using the option **Curve/Offset/Offset Curve** straight lines **BC** and **CD** are copied at 100cm towards the inside of the deltoid (fig. 113/2). Deltoid polygon is divided to special lengths using the option **Explode**, because the vertices have different heights, vertex **A** (1150cm), vertices **B** and **D** (250cm) and vertex **C** (550cm), which were moved in **Perspective** view using the option **Mini/Menu/Move** (fig. 113/3). Two opposite sides of the spatial deltoid **AD** and **BC** were used to model one HP using the option **Surface/Loft/Style: Normal** (fig. 113/4). Then, in **Perspective** view, using the option **Surface/Extrude Curve/Straight** vertical walls were modelled above the longer sides of the deltoid and parts above the HP were removed using the option **Mini/Menu/Trim** (fig. 113/5). In order to obtain the whole object, one HP with vertical walls was rotated and copied using the option **Array/Polar/Number of Items 3/360°** in **Top** view. The thickness of the shell was obtained by using the option **Solid/Offset**, and lanterns were modelled using the command **Surface/Surface from 3 or 4 Corner points** (fig. 113/6).

### HOMEWORK 9

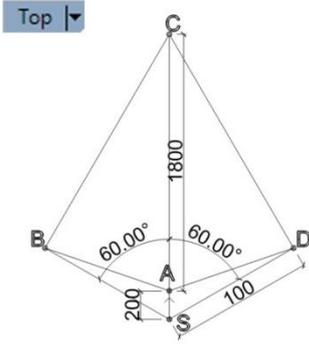
HOMEWORK 9 models an object generated of two hemispheres (fig. 115) in 7 steps, with given dimensions. That is why it is necessary to change measurement units from mm to cm using the option **File/Properties/Units**. In **Top** view, using the option **Solid/Sphere** one sphere was modelled, with the center **L** and radius  $r=1000\text{cm}$ , whose lower half was removed using the options **Surface/Plane/Cutting Plane** (fig. 115/1) and **Mini/Menu/Trim**. Then in **Front** view, the sphere was cut with yet another inclined plane through the center, at the angle of  $-60^\circ$  (fig. 115/2). The plane was moved from the center to the right for 500cm and the smaller part of the sphere was removed using the option **Mini/Menu/Trim** (fig. 115/3). Then the inclined plane was rotated into vertical position (fig. 115/4) and the whole object was generated using the option **Transform/Mirror** in relation with vertical plane (fig. 115/5).

## EXERCISE 9 – Rhinoceros: Model the given object using hyperbolic paraboloid

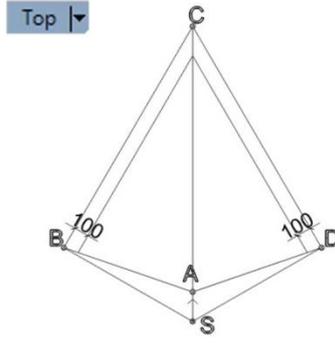
Procedure for constructing an object formed by joining three hyperbolic paraboloids from figure 113:

1. Set units in cm using the option **File/Properties/Units/Model units: Centimeters**;
2. Drawing the deltoid: Using the option **Curve/Polyline** in **Top** view, draw vertical line "SD" 1000cm long; Using the options **Mini Menu/Text + Curve/Point Object/Single Point** draw and mark bottom "S" and upper "D" point of line;
3. Rotate and copy the line "SD" around bottom point "S" at angle of  $120^{\circ}$  using the option **Transform/Rotate/Copy = Yes** and get line "BS"; Using the options **Mini Menu/Text + Curve/Point Object/Single Point** draw and mark point "B";
4. In **Top** view from point "S" draw vertical line "SC" in length 2000cm, using the option **Polyline** and rotate it at the angle of  $60^{\circ}$  using the option **Transform/Rotate/Copy = No** (*line "SC" should be bisector to angle "BSD"*); Use options **Mini Menu/Text + Curve/Point Object/Single Point** to draw and mark point "C";
5. In **Top** view draw and mark point "A" on line "SC" at 200cm from the point "S" using the options **Curve/Polyline, Mini Menu/Text + Curve/Point Object/Single Point**;
6. Using the option **Curve/Polyline** in **Top** view connect points "ABCD" and obtain a deltoid;
7. Using the option **Mini Menu/Explode** divide deltoid polygon to special lengths;
8. In **Top** view copy lines "BC" and "CD" for 100cm to deltoid interior using the option **Curve/Offset/Offset Curve**; Using the option **Mini Menu/Trim/Apparent Intersections=Yes** remove shorter parts of copied lines in relation to the length "SC" so that they intersect and in relation to length "AB" and "AD";
9. In **Front** view, in the direction of the axis z, raise points "B" and "D" for 250cm, point "C" for 550cm, and point "A" for 1150cm, using the command **Transform/Move**;
10. Using the option **Curve/Line/Single Line** in **Perspective** view, connect points and get lengths "AD" and "BC";
11. Using the option **Surface/Loft/Style: Normal** in the same view, connect opposite lengths "AD" and "BC" and model a hyperbolic paraboloid (*choose first "AD"*);
12. Using the option **Surface/Extrude Curve/Straight** in **Perspective** view, along the direction of axis z, from lines in horizontal xy plane, (*obtained with Offset option – step 9*), model vertical planes, heights of 600cm;
13. In **Perspective** view delete parts of vertical planes that are above HP using the option **Mini Menu/Trim**;
14. Using the option **Transform/Array Polar/Number of Items 3/360<sup>o</sup>** in **Top** view copy and rotate all surfaces, hyperbolic paraboloid and vertical planes, center of rotation is point "S";
15. Using the option **Surface/Surface from 3 or 4 Corner Points** in **Perspective** view, fill out the holes for lanterns;
16. Using the option **Solid/Offset**, give thickness of surfaces of your choice;

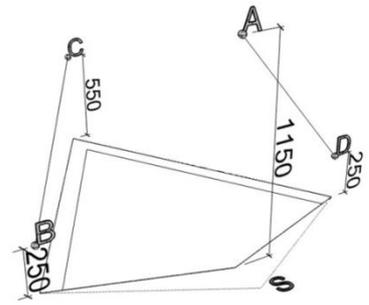
1. DRAWING DELTOID "ABCD" AND MOVING POINT "A"



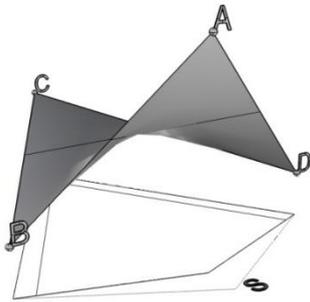
2. WITHDRAWING AND TRIMMING OF SIDES "BC" AND "CD"



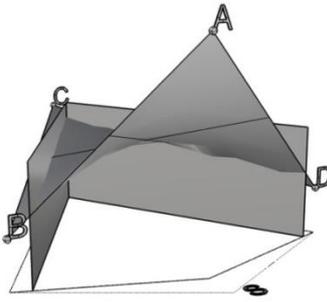
3. RAISING ALL POINTS TO CORRECT HEIGHT



4. MODELING OF HYPERBOLIC PARABOLOID



5. SETTING VERTICAL WALL PLANES



6. ROTATING AND COPYING SURFACE AND WALLS FOR THE FINAL MODEL

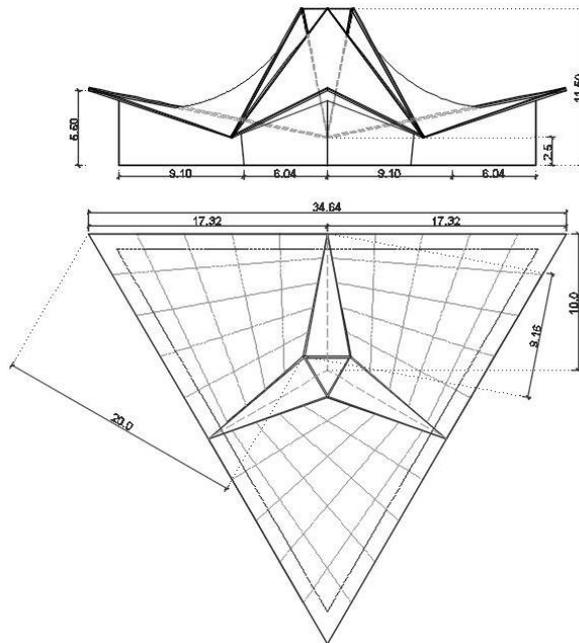
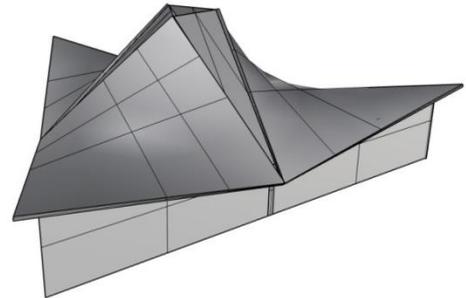


Figure 113 – Procedure for modelling hyperbolic paraboloid object above triangular basis (up) and top and front view (down)

Select a display mode ( Viewport=Active Mode=Wireframe): \_Mode=\_Ghosed

Display mode set to "Ghosed".

Command: |

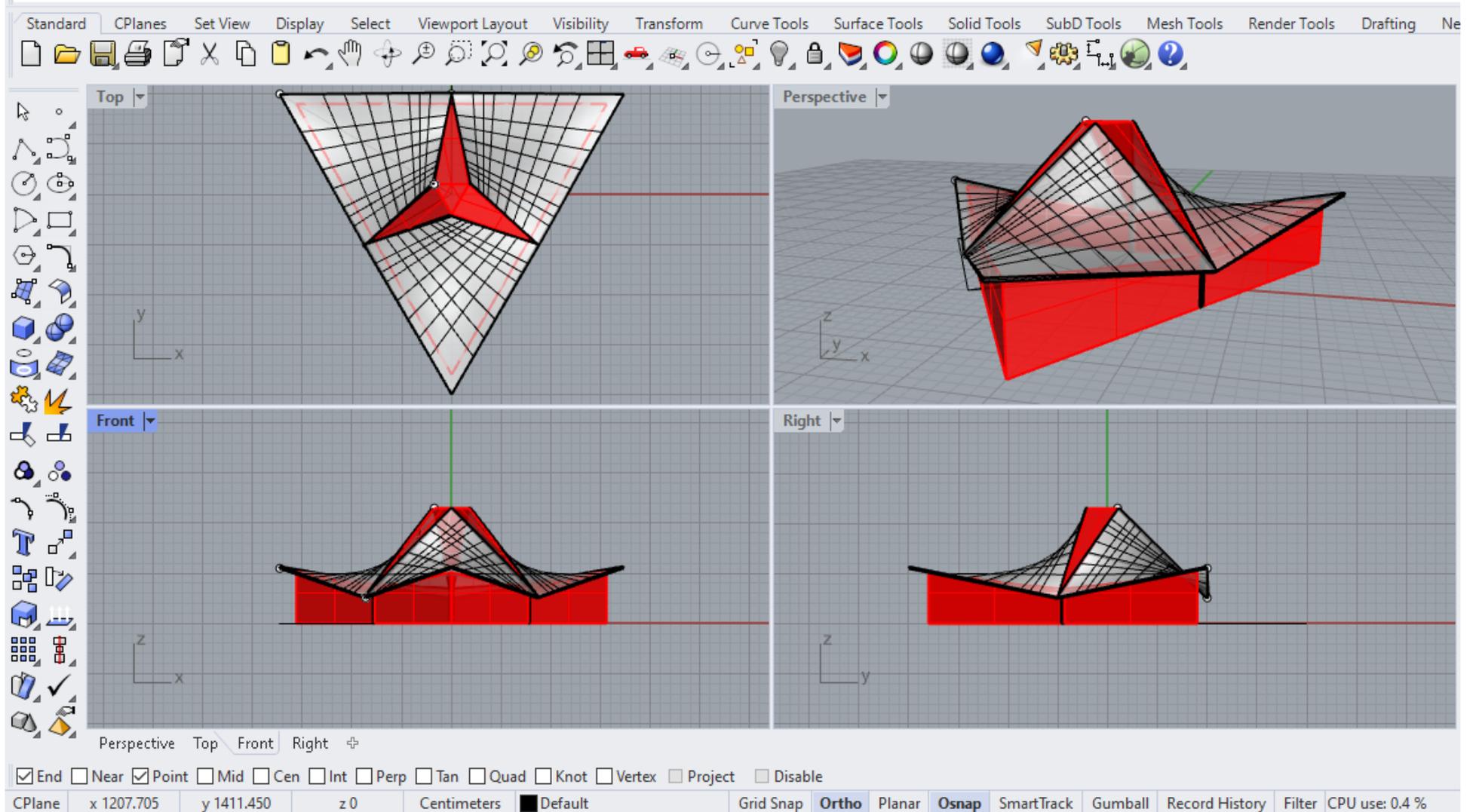
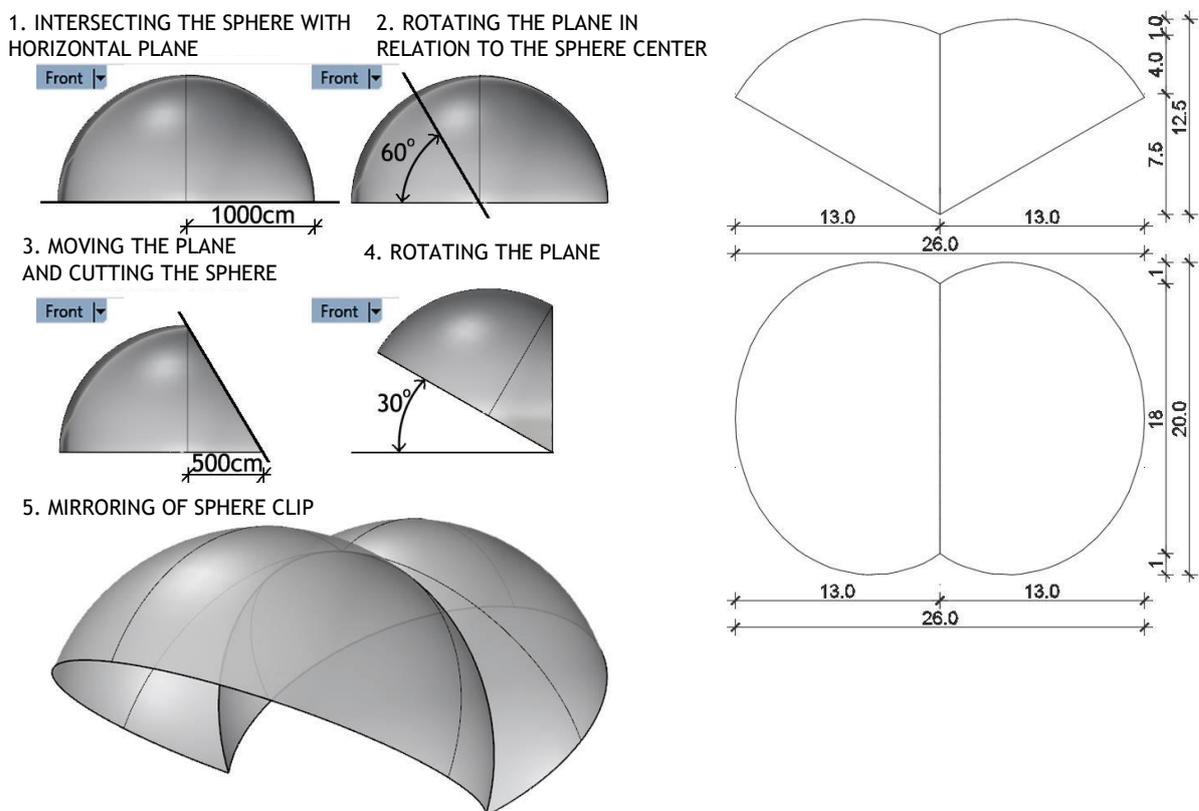


Figure 114 – Example Exercise 9

## HOMEWORK 9: Model the given object using the sphere

### Procedure for constructing spherical canopy from figure 115:

1. In **Top** view, draw a sphere using the option **Solid/Sphere/Center, Radius 1000cm**;
2. In **Front** view set a horizontal plane that intersects the sphere through center using the option **Surface/Plane/Cutting Plane**;
3. Using the option **Trim** remove the part of the sphere under the plane;
4. In **Front** view rotate the horizontal plane at the angle of  $-60^\circ$  in relation to the center of hemisphere and then move it 500cm to the right of the center in direction x in order to obtain the shape from figure 115;
5. Using the option **Trim** remove the smaller clip of the sphere;
6. In **Front** view rotate the clip of the sphere at  $-30^\circ$ ;
7. Using the option **Transform/Mirror** mirror the obtained clip of the sphere;



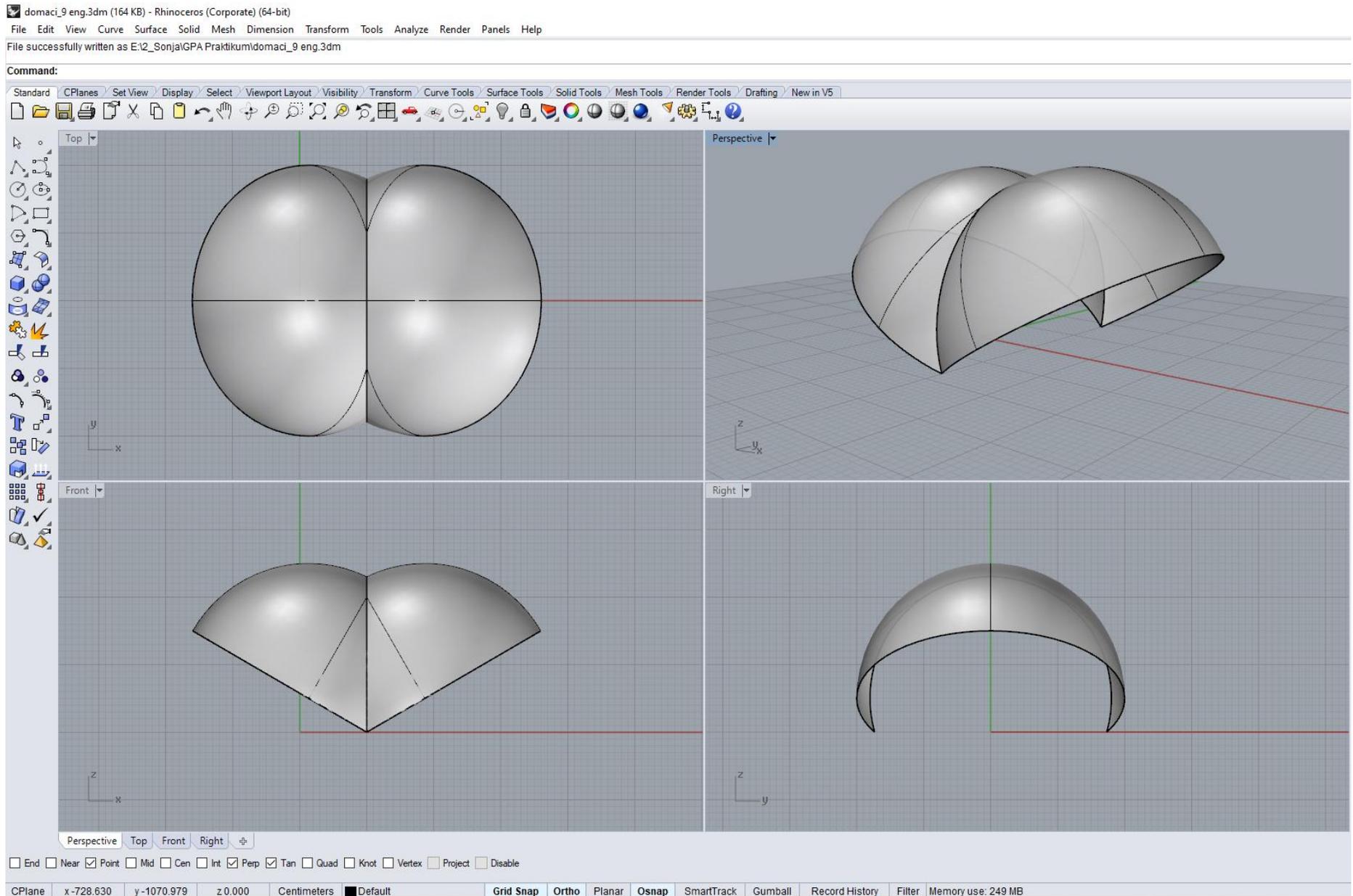


Figure 116 – Example Homework 9

Figures 117 and 118 show representative examples of the use of hyperbolic paraboloid and sphere in architecture.



Figure 117 – HP, Church of San Jose Obrero, Monterrey, Mexico, arch. F. Candela, E. Mor, F. Karman, year 1959



Figure 118 – Sphere, Eden project, Cornwall, UK, arch. N. Grimshaw, and A. Hunt Associates, year 2000

# UNIT 10 – RULED SURFACES WITH TWO GENERATRIX SYSTEMS: ONE-SHEET ROTATIONAL HYPERBOLOIDS

## SPECIAL ROTATIONAL SURFACE: SPHERE (BALL)

### ONE-SHEET ROTATIONAL HYPERBOLOIDS

One-sheet rotational hyperboloid (HOS) is one of the non-developable, ruled surfaces of II degree, with two generatrix systems, because all three **directrices** (guidelines) are lines, two in finiteness and one infinitely distant (fig. 119 and 120). Since HOS can be generated by rotation of one curve around axis, they also belong to the rotational surfaces.

One-sheet rotational hyperboloid is generated in two ways:

- By rotation of two generatrix systems  $g_1$  and  $g_2$  around the other line  $o$ , with which they are skew (fig. 119).
- By rotation of the hyperbola  $h$  around its minor axis  $o$  (fig. 120). Each point of hyperbola  $h$  makes a circle of rotation, which is perpendicular to the axis of rotation  $o$ .

These circles of rotation are called **parallels**. The smallest circle of rotation, of vertex of the hyperbola, point  $T$ , is a restriction (throat) parallel. With its rotating, hyperbola asymptotes determine asymptotic cone of hyperboloid, with the top  $H$ , at the center of the surface (fig. 119 and 120). Figure 121 shows two variants of one-sheet rotational hyperboloid, when their bases circles are not with equal radius.

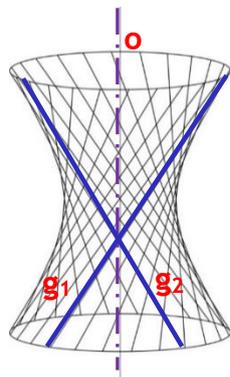


Figure 119 – One-sheet rotational hyperboloid rotation of generatrices  $g_1$  and  $g_2$  around line  $o$

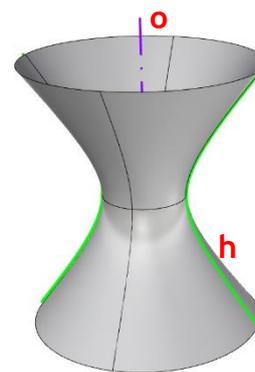


Figure 120 – One-sheet rotational hyperboloid rotation of hyperbola  $h$  around axis  $o$

a)



b)



Figure 121 – One-sheet rotational hyperboloid, two variants when bases are not with equal radius

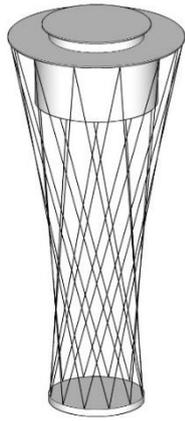


Figure 122 – HOS, El Prat Airport, Spain, arch. R. Bofill Levi, year 1996

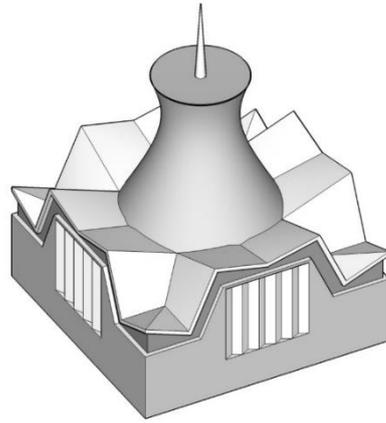


Figure 123 – HOS, Sacred Heart Basilica, arch. Herb, Kutter and Sarger, year 1966

## SPHERE (BALL)

### Sphere planar cross sections

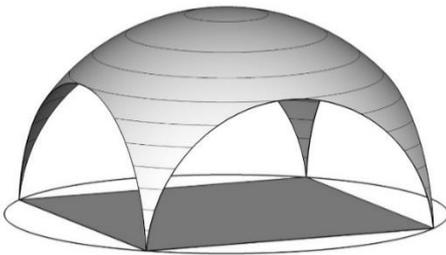


Figure 124 – Hemisphere over square basis

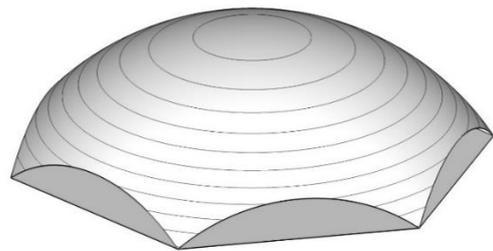


Figure 125 – Spherical calotte over hexagonal basis

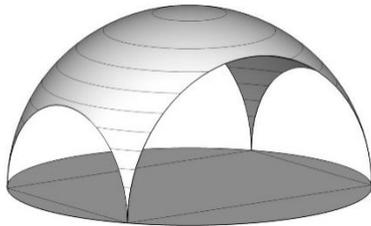


Figure 126 – Hemisphere over rectangular basis

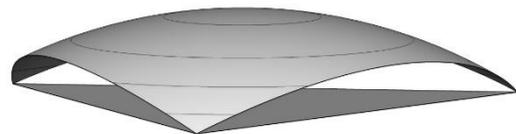


Figure 127 – Spherical calotte over triangular basis

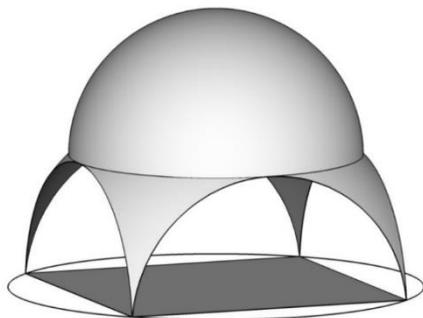


Figure 128 – Dome over pendentives (spherical triangles)

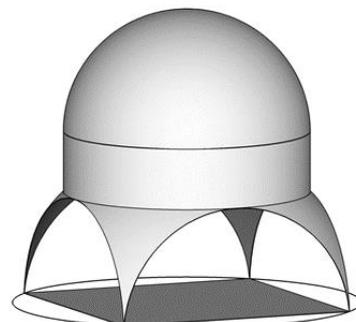


Figure 129 – Dome over tambour (cylinder) over pendentives

# MODELLING ONE-SHEET ROTATIONAL HYPERBOLOID'S OBJECT AND SPHERE TRIANGULAR SLICE IN THE COMPUTER SOFTWARE RHINOCEROS

## EXERCISE 10

EXERCISE 10 showed how to construct an object from one-sheet rotational hyperboloid (HOS), Cathedral Brasilia, architect O. Niemeyer (fig. 130), in 15 steps, with given dimensions. That is why it is necessary to change measurement units, from mm to cm, using the option **File/Properties/Units**. Firstly, a hyperbola is drawn in vertical plane, in **Front** view, using the option **Curve/Hyperbola/Center, Focus**. Hyperbola is given by the center and focus determined by points of a polyline, using the options **Single Point** and **Text Object, A, B, C** and **D** (it is not necessary to choose drawing of the hyperbola with both branches, fig. 130/1). Then, in the same view, using the option **Surface/Revolve/360°**, where the axis of the surface passes through the center of the hyperbola, point **A**, a basic one-sheet hyperboloid (HOS) is modelled. The surface of the HOS is cut with a horizontal plane, at the height of 1700cm, so that it would not be symmetrical, and the smaller part is removed using the option **Mini/Menu/Trim** (fig. 130/2). At the same time, a part of the hyperbola above the cutting plane was also removed, and the new uppermost point of the hyperbola was marked with **E**, using the options **Single Point** and **Text Object**. Throat circle was determined, using the option **Curve/Curve from Object/Intersection**, which contains vertices of the hyperbolas (horizontal plane passes through the points **A** and **B**) and divided into 32 parts, using the option **Curve/Point Object/Divide Curve by Number** (fig. 130/3). In order to generate the carriers (pillars) of the object, the middle point of the shortened basic hyperbola, activated by the option **F10**, is moved horizontally outside the surface for 100cm, in **Perspective** view. Another hyperbola is needed, determined by the intersection of the inclined plane passing through the points on the bases of the main hyperbola (**D** and **E**) and the first adjacent point, determined by the division into 32 parts, on the throat circle (in relation to the point in the vertical plane through **DE**). That is why the vertical plane through the points **D** and **E** is rotated using the option **Transform/3D Rotate** (rotation axis is line **DE**) and its intersection with HOS is determined (fig. 130/4). One half of the pillar surface is generated using the option **Surface/Loft**, where shortened basic hyperbola and intersected hyperbola with the inclined plane were chosen. Then the surface was mirrored, using the option **Transform/Mirror**, in order to obtain the whole carrier (fig. 130/5). Using the command **Join**, 2 surfaces are merged into one, which was radially copied by using the option **Transform/Array/Polar/Number of Items 16/Angle 360°**. It was necessary once again to cut the surface of the HOS with lower horizontal plane (at the height of 1400cm), so that the pillars are outside the surface, and then close it on both sides using the option **Cup** (fig. 130/6).

## HOMEWORK 10

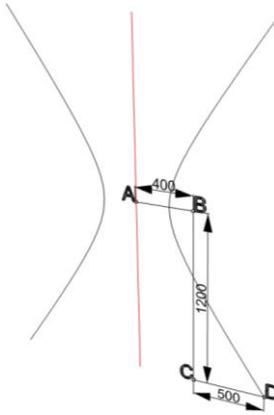
For HOMEWORK 10 it is necessary to model a slice of the sphere over a triangular basis (fig. 132), in 6 steps, with given dimensions. That is why it is necessary to change measurement units, from mm to cm, using the option **File/Properties/Units**. Firstly, an equilateral triangle is drawn, side  $a=2000\text{cm}$ , in **Top** view, using the option **Curve/Polygon/Edge**, the heights determined (precisely with **Osnap/Perpendicular**) as well as the orthocenter (fig. 132/1). Then, a sphere is modelled, using the option **Solid/Sphere/Center, Radius**, with radius  $r=2000\text{cm}$ , with the center in the orthocenter of the equilateral triangle, in **Perspective** view (fig. 132/2). Using the option **Surface/Extrude Curve/Straight** a vertical three-sided prism is modelled (basis is the equilateral triangle, fig. 132/3), in the same view. In the end, all parts of the sphere outside the prism should be removed, using the option **Mini/Menu/Trim** (fig. 132/4), and then the prism shell is removed (fig. 132/4 and 132/5).

## EXERCISE 10 – Rhinoceros: Model the given object using one-sheet rotational hyperboloid

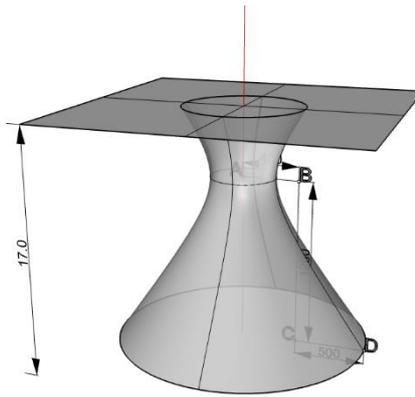
Procedure for constructing one-sheet hyperboloid's object from figure 130:

1. In **Front** view, draw auxiliary lines for constructing hyperbola with **Polyline** option: horizontal "AB" 600cm (*from left to right*), vertical "BC" 1100cm (*down to*) and horizontal "CD" 400cm (*to the right*). Lines are drawn one after the other;
2. In **Front** view, draw hyperbola with **Curve Hyperbola/Center, Focus** option by selecting first point "A", then "B" and "D";
3. In **Front** view, use the option **Surface/Revolve/360°** to model one-sheet rotational hyperboloid around vertical axis which passes through "A" point (*center of rotational hyperboloid*);
4. In **Front** view, set the horizontal plane through hyperboloid at height of 1700cm from the bottom basis, with **Surface/Plane/Cutting Plane** option;
5. In **Front** view by **Mini Menu/Trim** option cut and discard part of hyperboloid and hyperbola that are above the cutting plane; Mark new upper endpoint of the hyperbola with the "E" using the option **Single Point, Text Object**;
6. In **Front** view, set the new horizontal cutting plane at height of throat circle (*at height of points "A" and "B"*) using the option **Surface/Plane/Cutting Plane**, then find hyperboloid intersection with that plane using the option **Curve/Curve from Object/Intersection**;
7. In **Perspective** view, move horizontally center point of hyperbola (*which is activated by F10 option*), 100cm to the right;
8. Divide the throat circle into 32 parts using the option **Curve/Point Object/Divide Curve by Number** in **Perspective** view;
9. In **Perspective** view, create a vertical plane through "D" and "E" hyperbola points with **Surface/Plane/Cutting Plane** option (*selection of rotational hyperboloid and setting points "D" and "E"*);
10. In the same view, rotate vertical plane using the option **Transform/3D Rotate** so that the axis of rotation is defined by points "D" and "E" of hyperbola, and rotating plane passes through the first adjacent point on throat circle;
11. In **Perspective** view, find intersection curve of hyperboloid and oblique plane using the option **Curve/Curve from Object/Intersection**;
12. In **Perspective** view, create surface (carrier) using hyperbola (*generated in step 7*) and intersection curve (*generated in step 8*) with **Surface/Loft** option and mirror it using the option **Transform/Mirror** in relation to points "D" and "E". Then join both surfaces in one unit;
13. In **Top** view, rotate and copy generated carriers using the option **Transform/Array/Polar 16, Angle 360°**, center of rotation is point "A" (*center of hyperboloid*);
14. In **Front** view, set horizontal intersection plane at height of 1400cm from bottom basis using the option **Surface/Plane/Cutting Plane** and cut off part of hyperboloid above that plane using the **Trim** option;
15. Close hyperboloid from upper and lower side using the **Cap** option;

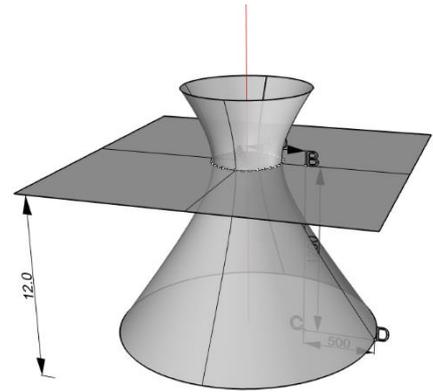
1. DRAWING HYPERBOLA



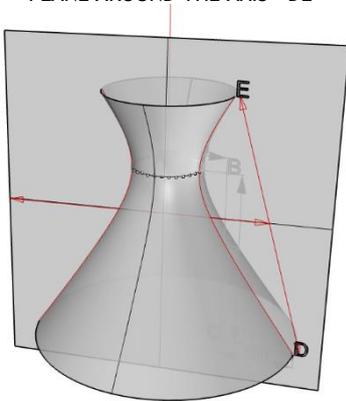
2. CUTTING HYPERBOLOID WITH PLANE AT HEIGHT OF 17M



3. MOVING ONE POINT OF HYPERBOLA



4. ROTATION OF VERTICAL PLANE AROUND THE AXIS "DE"



5. MODELING OF ONE CARRIER OF OBJECT



6. FINAL MODEL OF OBJECT

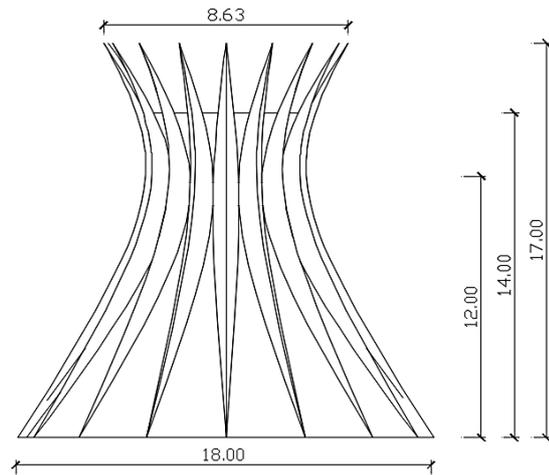
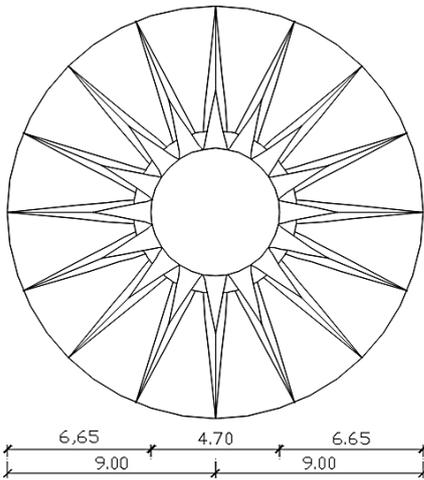
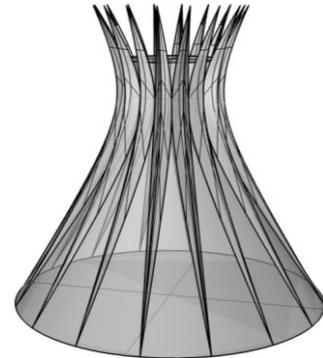


Figure 130 – Procedure for modelling one-sheet rotational hyperboloid's of object (up) and top and front view (down)

Command: |

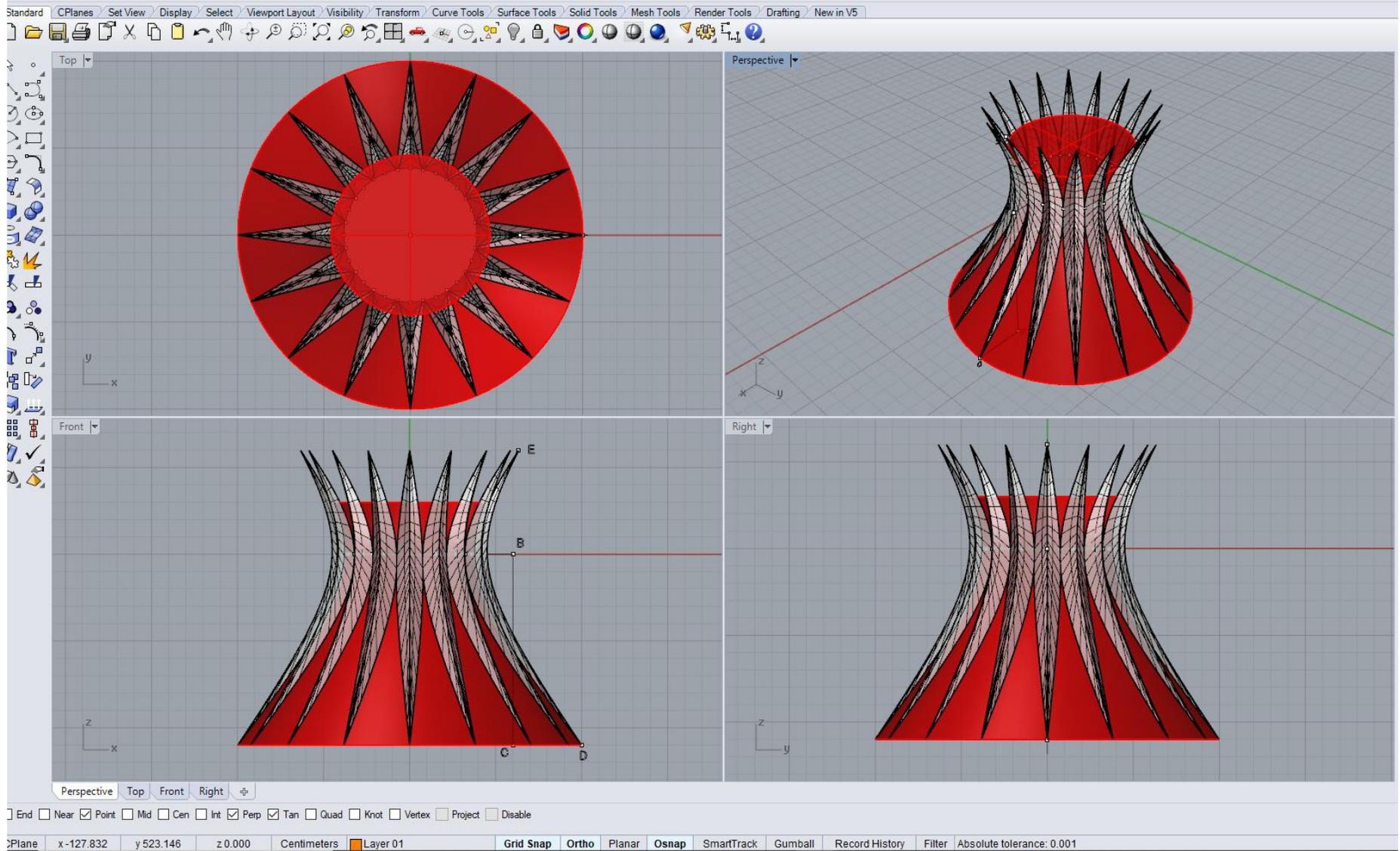


Figure 131 – Example Exercise 10

## HOMEWORK 10: Model the given object using the sphere

### Procedure for constructing sphere slice from figure 132:

8. In **Top** view, draw an equilateral triangle of 2000cm side using the option **Curve/Polygon/Edge**;
9. Draw the height of the triangle and determine the orthocenter (*plug in **Osnap/Perpendicular***) in the same view;
10. Make sphere model by using command **Solid/Sphere/Center, Radius 2000cm**, with the center in the triangle orthocenter;
11. Using the option **Surface/Extrude Curve/Straight** make vertical surfaces (prism) from the triangle lines, height above the highest point of sphere in **Perspective** view;
12. Using the **Trim** option, remove part of sphere that is above the triangular prism in the same view;
13. Delete the vertical planes of prism.

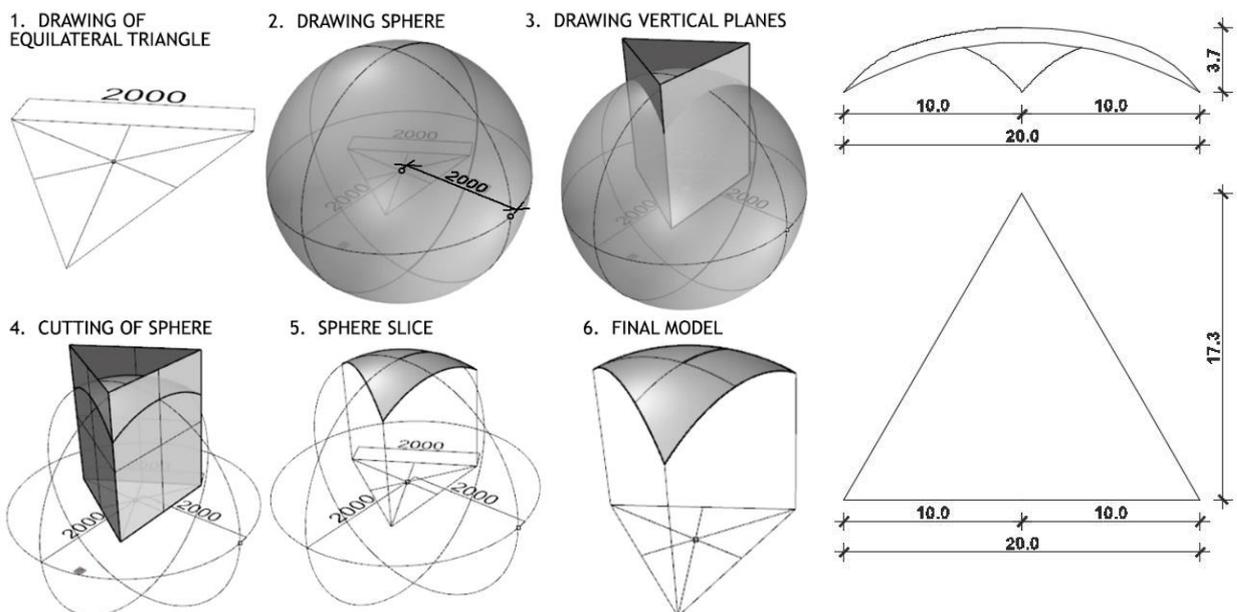


Figure 132 – Procedure for modelling sphere slice (left) and top and front view (right)

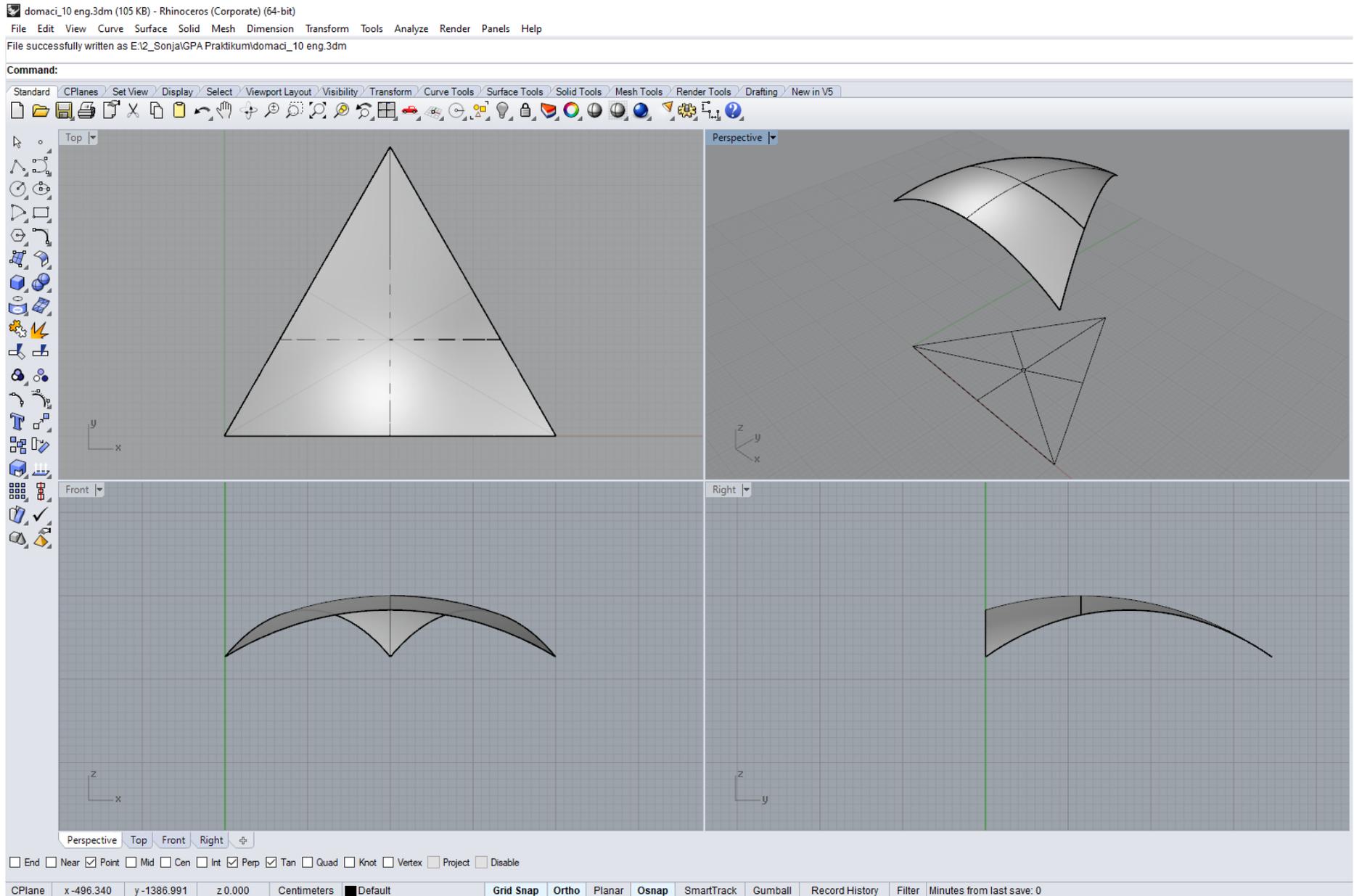


Figure 133 – Example Homework 10

Figures 134 and 135 show representative examples of the use of revolving hyperboloid of one sheet and sphere in architecture.



Figure 134 – HOS, Cathedral Brasilia, Brazil, arch. O.Niemeyer, year 1970



Figure 135 – Sphere, Opera House, Sydney, Australia, arch. J. Utzon and O. Arur, year 1973

# UNIT 11 – GENERAL SURFACES (QUADRICS): ELLIPSOIDS AND PARABOLOIDS

## ELLIPSOIDS

Ellipsoids (fig. 136 and 137) belong to general surfaces of II degree (quadrics). Possible intersections of ellipsoids are real or imaginary ellipses and circles.

Ellipsoids are divided into:

**Triaxial** – general surface of II degree, which has three mutually perpendicular axes, of different lengths, lying in three mutually perpendicular planes, cutting it along the ellipses (fig. 136). **Directrix** (guideline) **d**, of these surfaces is one of ellipse and the **generatrix** **g**, is one of the other two ellipses.

**Rotational** – general surface of II degree, which has one main axis and  $\infty^1$  of couples of the other two axes (fig. 137). **Directrix** (guideline) **d**, of these surfaces is a circle, and the **generatrix** **g**, is an ellipse. It is generated by rotating the ellipse around one of its axes. If the rotation axis is the longer axis of the ellipse, elongated rotating ellipsoid is generated, and if the rotating axis is the shorter one, a flattened rotating ellipsoid is generated.

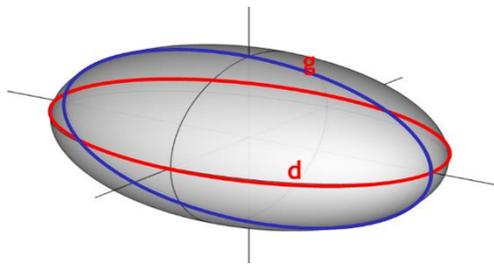


Figure 136 – Triaxial ellipsoid

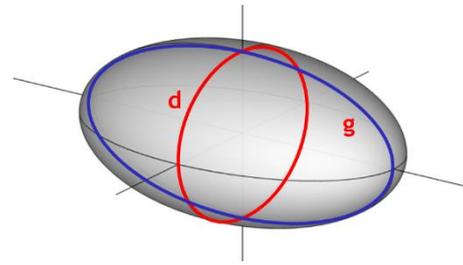


Figure 137 – Rotational ellipsoid

## PARABOLOIDS

Paraboloids (fig. 138 and 139) belong to general surfaces of II degree (quadrics). Possible intersections of paraboloids are real or imaginary ellipses, circles or parabolas.

Paraboloids are divided into:

**Rotational** – general surface of II degree, which has one axis, lying in the parabola plane and the planes perpendicular to axis, cutting it along the circles (fig. 138). **Directrix** (guideline) **d**, of these surfaces is a circle and the **generatrix** **g**, is a parabola. It is generated by rotating the parabola around its axis.

**Elliptic** – general surface of II degree, which has one axis, lying in the parabola plane and the planes perpendicular to axis, cutting it along the ellipses (fig. 139). **Directrix** (guideline) **d**, of these surfaces is an ellipse and the **generatrix** **g**, is a parabola. These surfaces belongs to special translational surfaces.

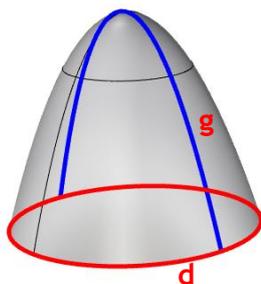


Figure 138 – Rotational paraboloid

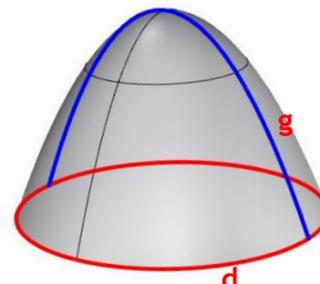


Figure 139 – Elliptic paraboloid

# MODELLING TRIAXIAL ELLIPSOID'S OBJECT AND ROTATIONAL PARABOLOID'S OBJECT IN THE COMPUTER SOFTWARE RHINOCEROS

## EXERCISE 11

EXERCISE 11 showed how to construct a cross-vault from the triaxial ellipsoid (fig.140), in 12 steps, with given dimensions. That is why it is necessary to change measurement units, from mm to m, using the option **File/Properties/Units**. Firstly, it is necessary to model a triaxial ellipsoid, whose horizontal ellipse is placed inside a rectangle, sides 20x10m, which are the lengths of the horizontal axes of the ellipsoid, and the vertical axis is length of 3m. That is why it is necessary first to draw a basic rectangle in **Top** view, with sides 20m and 10m, using the option **Curve/Rectangle**, and then model the ellipsoid using the option **Solid/Ellipsoid/From Center**, with the previously stated axes lengths, whose center coincides with the center of the rectangle, precisely by turning on the option **Osnap/Center** (fig. 140/1). Ellipsoid is cut by vertical planes through the sides of the rectangle, so it is necessary to enlarge the starting ellipsoid by using **Transform/Scale**, so that the horizontal ellipse is encircled around the rectangle 20x10m. In order to perform the option **Transform/Scale** the simplest way is to firstly draw a diagonal of the rectangle in **Perspective** view, and then determine its intersection with the ellipsoid by using **Curve/Curve from Object/Intersection** (diagonal of the rectangle is the longer axis of the ellipse, obtained in the intersection of the vertical plane through the diagonal). The new bigger ellipsoid has the longer axis of the intersecting ellipse equal to the diagonal of the basic rectangle (fig. 140/2). Then, in the same view, intersecting vertical planes are placed through the sides of the rectangle and a horizontal plane through the center of the ellipsoid, using the option **Surface/Plane/Cutting Plane** and its lower and outer parts are removed using the option **Trim** (fig. 140/3). Vertical planes above the shorter sides of the rectangle are copied and rotated using the option **Transform/3Drotate/Copy=Yes** at the angle of  $20^\circ$  (rotation axis is the appropriate side of the rectangle), in order to obtain overhangs on the vault (fig. 140/4). The thickness of the shell  $d=0.1\text{m}$  is obtained by using the option **Solid/Offset**. In order to obtain the whole cross-vault, the option **Transform/Rotate** is used, (rotation center is the center of the basic rectangle, rotation angle is  $90^\circ$ , fig. 140/5). Then, in order to remove inner parts of vertical planes, firstly the planes are separated from the ellipsoid surface using the option **Split**, and then removed using the option **Delete**.

## HOMEWORK 11

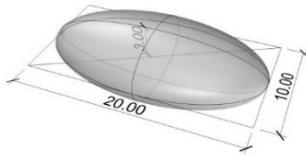
HOMEWORK 11 is to model an object from rotational paraboloid over a hexagonal basis (fig. 142), in 10 steps, with given dimensions. That is why it is necessary to change measurement units, from mm to m, using the option **File/Properties/Units**. Parabola is drawn using the option **Curve/Parabola/Vertex, Focus** (choose to mark vertex and focus) in the **Front** view, because the axis needs to be vertical, length 15m, and the tendon of the parabola is 60m ( $1/2=30\text{m}$ , fig. 142/1). Then the rotational paraboloid is modelled by rotating the parabola around the vertical axis using the command **Surface/Revolve  $360^\circ$** , in **Perspective** view (fig. 142/2). In the basis circle of the paraboloid, radius  $r=30\text{m}$  in **Top** view, a horizontal hexagon is placed, using the option **Curve/Polygon/Center, Radius**, whose center coincides with the center of the basis circle. Using the option **Curve/Offset/Offset, Curve** a new hexagon is placed inside the main one, with the distance of 6m, inwards. From those hexagons, using the option **Surface/Extrude Curve/Straight** two hexagonal prisms are modelled (fig. 142/3), with different heights,  $h=7\text{m}$  (bigger hexagon) and  $h=15\text{m}$  (smaller hexagon), in **Perspective** view. It is necessary to separate the paraboloid from vertical planes using the option **Split**. Firstly, the parts of the paraboloid outside the bigger hexagon are removed, then the upper part of the shell is lifted for 2m in **Perspective** view, using the option **Transform/Move/Vertical=Yes** (fig. 142/4). It is necessary to remove all parts of vertical planes above the paraboloid using the option **Mini Menu/Trim** and set the thickness of the shells (fig. 142/5). If desired, a door can be placed on the object and glass surfaces marked.

## EXERCISE 11 – Rhinoceros: Model the given object using triaxial ellipsoid

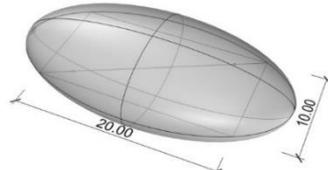
Procedure for constructing object from triaxial ellipsoid from figure 140:

1. Using **Curve/Rectangle** draw a basic rectangle base dimensions 20x10m in **Top** view;
2. In **Perspective** view, make a model of triaxial ellipsoid whose lengths of x and y axes are equal to sides of rectangle in base, with the height of vertical axis 3m using the option **Solid/Ellipsoid/From Center** (*turn on **Osnap/Cen***);
3. Draw diagonal of the rectangle using the **Curve/Polyline** option in **Perspective** view;
4. In **Perspective** view, find the intersection of the diagonal of rectangle and ellipsoid using the option **Curve/Curve from Object/Intersection**;
5. In **Perspective** view, enlarge the ellipsoid using the option **Transform/Scale** using the length of the axis of intersected curve (*from the center of the ellipsoid to the intersection point of solid with the rectangle diagonal so that the intersected ellipse axis is equal to the diagonal of the basic rectangle*);
6. In **Perspective** view, set the vertical cutting planes of ellipsoid over all 4 rectangular sides, and in **Front** view one horizontal plane through the ellipsoid center using the option **Surface/Plane/Cutting Plane**;
7. In **Perspective** view, rotate outwards two vertical planes over the shorter sides of the rectangle at an angle of  $20^{\circ}$  using the option **Transform/3Drotate/Copy=Yes** (*so that the rotation axis is the appropriate rectangle side, angle  $+20^{\circ}$  and  $-20^{\circ}$* );
8. In **Perspective** view, use the option **Trim** to cut a part of the ellipsoid (*first using horizontal, then inclined and vertical planes so that only the middle part remains*);
9. In **Perspective** view, reject the parts of vertical planes under and over the object using the option **Trim** (*first the vertical planes must be cut by the horizontal one, then vertical planes mutually and then vertical planes by the ellipsoid*);
10. In **Perspective** view, set the thickness of the shell  $d=0.1\text{m}$  using the option **Solid/Offset** and delete the horizontal plane;
11. In **Perspective** view, rotate and copy the entire object (*ellipsoid with vertical planes*) at the angle of  $90^{\circ}$  using the option **Transform/Rotate** (*center of the rotation is the center of basic rectangle*);
12. Delete all parts inside the object in **Perspective** view. Using the option **Mini Menu/Split** separate the vertical planes, and then **Delete** parts of vertical planes inside the object.

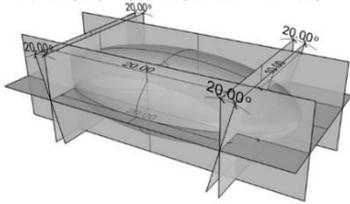
1. MODELLING OF ELLIPSOID WRITTEN INSIDE A RECTANGLE



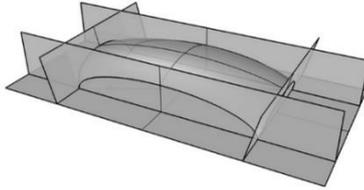
2. ENLARGING THE ELLIPSOID OVER THE RECTANGLE DIAGONAL



3. FORMING OF VERTICAL AND INTERSECTION INCLINED PLANES



4. CUTTING OF ELLIPSOID BY ALL PLANES



5. FINAL MODEL OF THE OBJECT

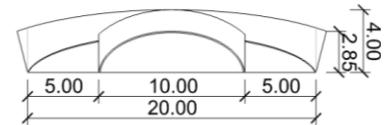
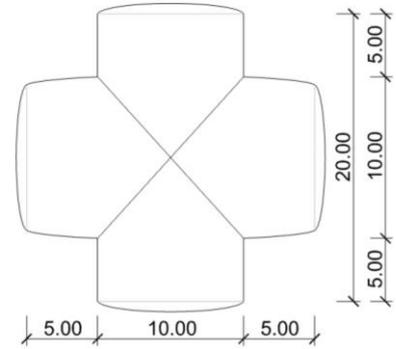
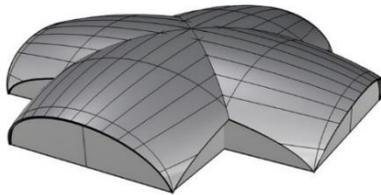


Figure 140 – Procedure for modelling of the ellipsoid's object (left) and top and front view (right)

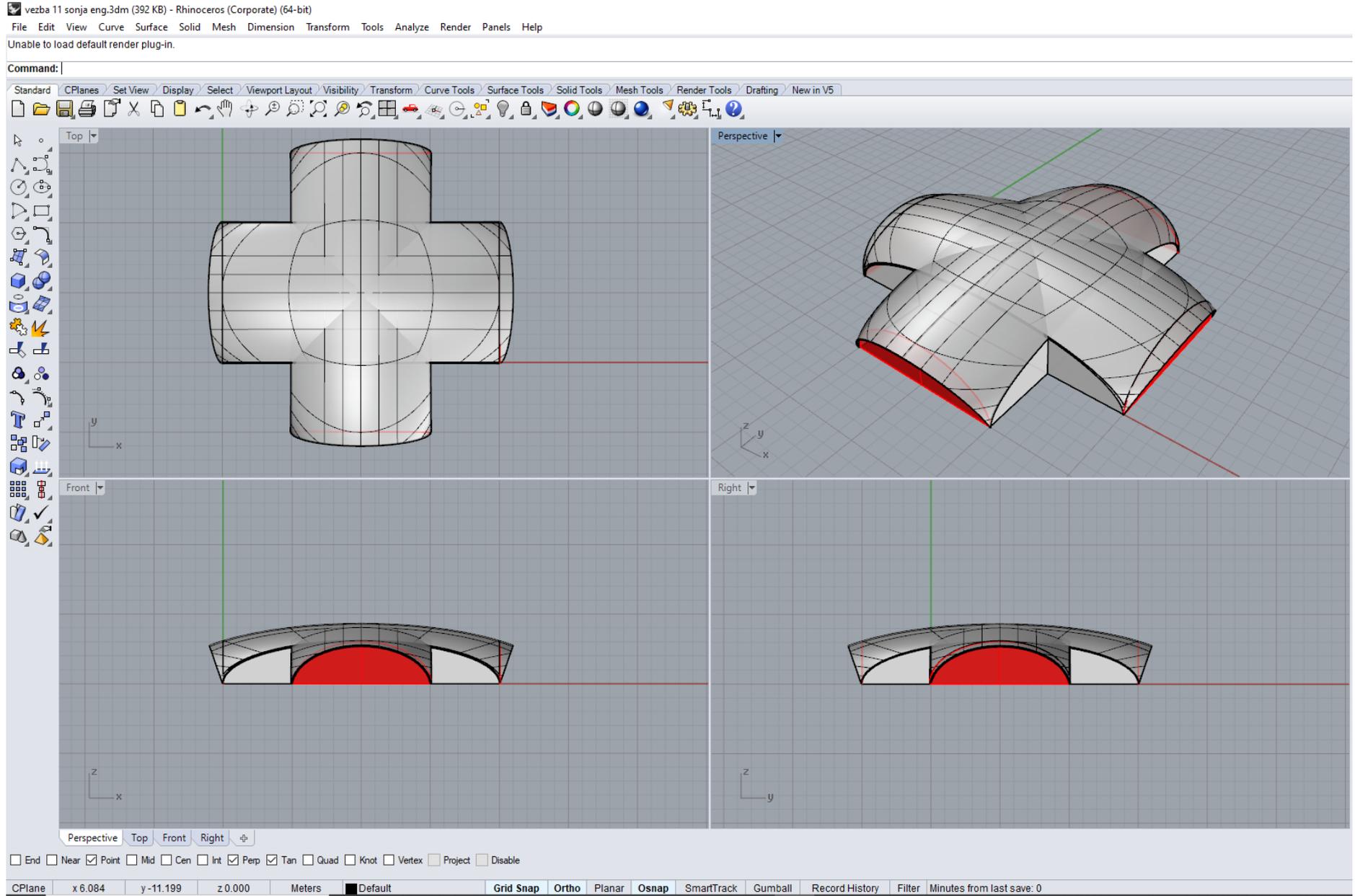


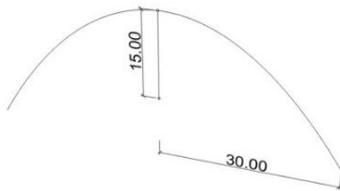
Figure 141 – Example Exercise 11

## HOMEWORK 11: Model the given object using rotational paraboloid

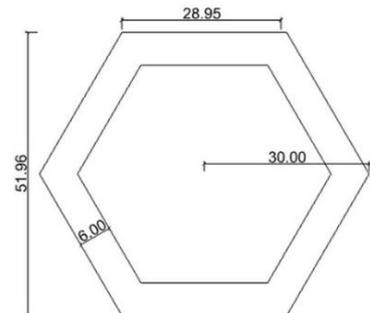
Procedure for constructing object from rotational paraboloid from figure 142:

1. In **Front** view draw a parabola using **Curve/Parabola/Vertex, Focus** with vertical axes of 15m and half tendon of 30m;
2. In **Perspective** view make a model of paraboloid using **Surface/Revolve 360°** so that the rotation axis is the parabola axis;
3. In **Perspective** view draw a hexagon with the center in the center of paraboloid base using the option **Curve/Polygon/Center, Radius** with 30m radius;
4. In **Top** view offset hexagon for 6m inwards using the option **Curve/Offset/Offset, Curve**;
5. In **Perspective** view use the option **Surface/Extrude Curve/Straight** to extrude the bigger polygon for 7m, and the smaller one for 15m;
6. In **Perspective** view use the option **Mini Menu/Split** to divide the paraboloid with vertical polygonal planes;
7. In **Perspective** view delete the parts of paraboloid which are outside the plane of the bigger hexagon using **Delete**;
8. In **Perspective** view use the option **Transform/Move/Vertical=Yes**, to lift the upper part of the paraboloid shell for 2m;
9. In **Perspective** view use the option **Mini Menu/Trim** to cut off polygonal walls above the parts of the paraboloid
10. *Arrange the shell, doors and windows as needed;*

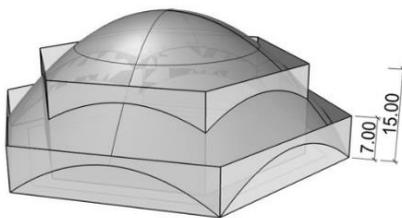
1. DRAWING OF THE PARABOLA



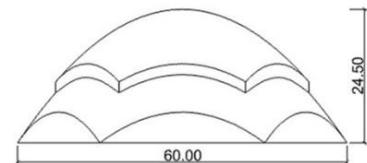
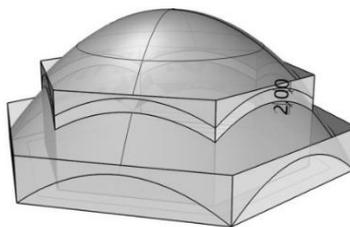
2. MODELLING THE PARABOLOID BY ROTATING THE PARABOLA



3. SETTING VERTICAL PLANES ABOVE BOTH HEXAGONS



4. LIFTING THE UPPER PART OF THE SHELL FOR 2M



5. FINAL MODEL OF THE OBJECT



Figure 142 – Procedure for modelling of the paraboloid's object (left) and top and front view (right)

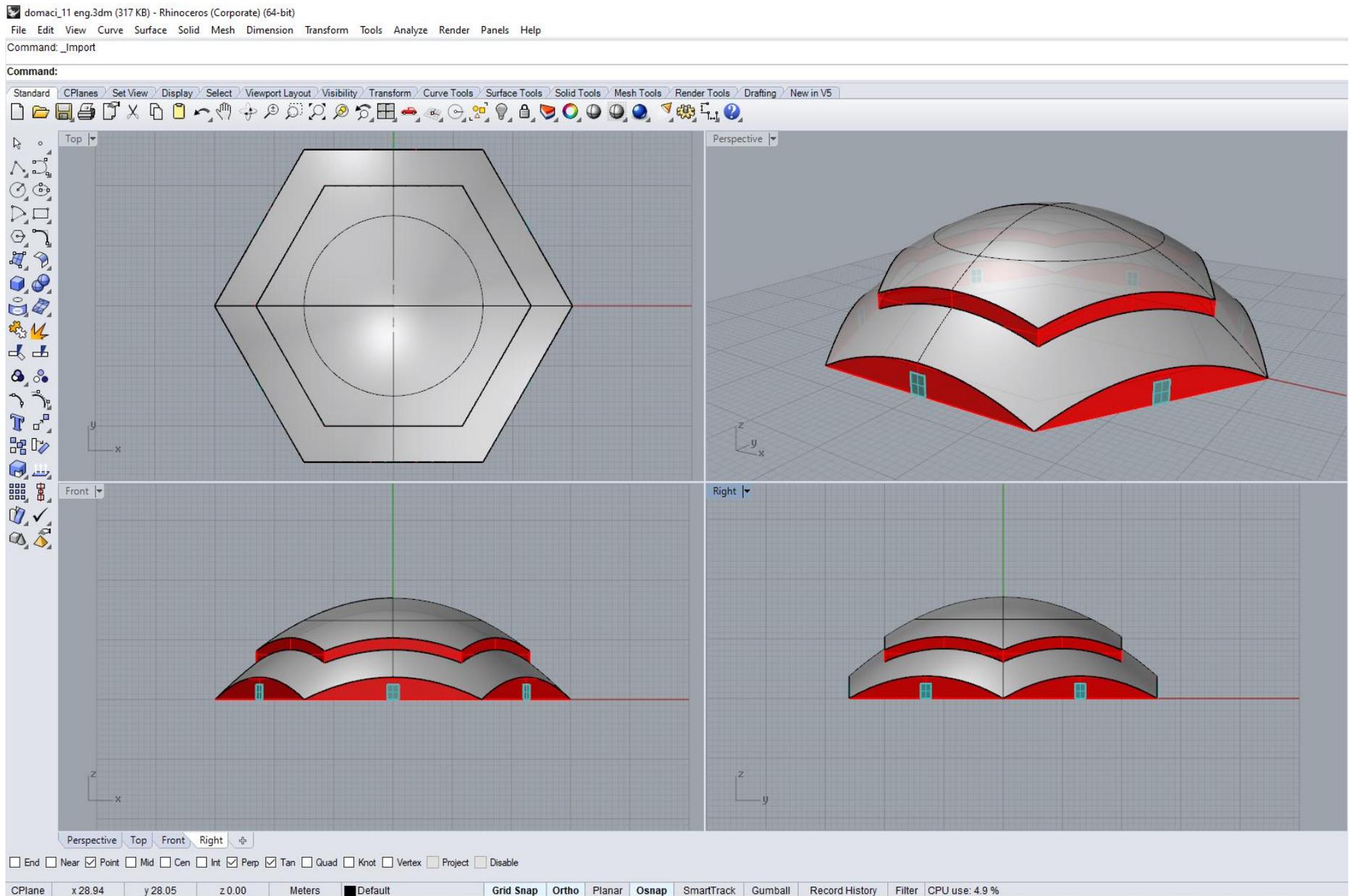


Figure 143 – Example Homework 11

Figures 144 and 145 show representative examples of the use of triaxial ellipsoid and rotational paraboloid in architecture.



Figure 144 – Triaxial ellipsoid, Infosys administration building, Pune, India, arch. H. Contractor, year 2008



Figure 145 – Rotational paraboloid, The first Christian church, Oklahoma, USA, arch. R. Connor and F. Sullivan, year 1956

# UNIT 12 – SPECIAL ROTATIONAL SURFACES: TORUS AND GENERAL TRANSLATIONAL SURFACES

## ROTATIONAL SURFACES

Rotational surfaces are generated by rotation of a planar or spatial curve around an axis (straight line). The curve is called **generatrix g**, and the line is the axis of the rotational surface. Each point of the generatrix forms the circle of the rotation perpendicular to the axis of the surface. Those circles of rotation are called parallels and they are the **directrices d** (guidelines). Each plane through the surface axis cuts it along the planar curve, which is symmetrical in relation to the axis and is called meridian. Meridian is the **generatrix** of the surface.

Rotational surfaces are divided into:

1. **general rotational surfaces** (quadrics) - ellipsoids, paraboloids, hyperboloids,
2. **special rotational surfaces** - spheres, cones, cylinders, toruses (fig. 146)
3. **general type of rotational surfaces** (fig. 147)

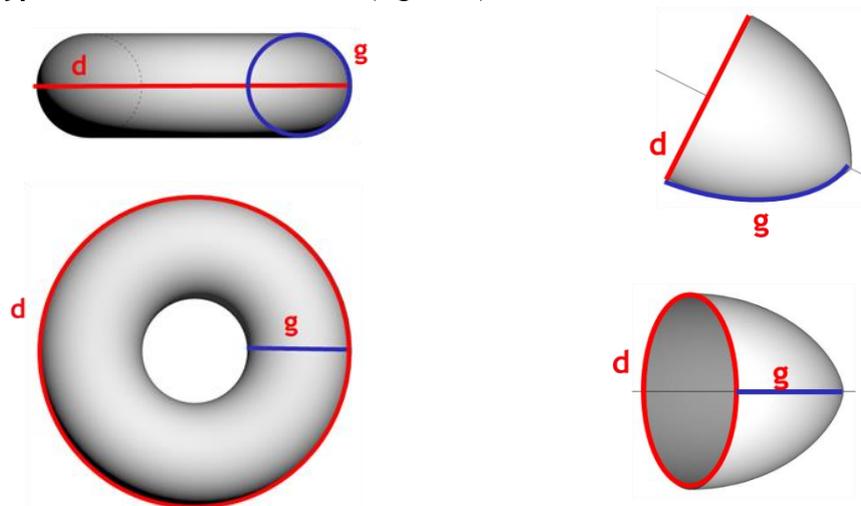


Figure 146 – Special rotational surfaces, tori      Figure 147 – General type of rotational surface

## TORUS

Torus (fig. 146) belongs to special rotational surfaces of IV degree, obtained when a circle rotates around an axis, **a**, which is in the plane of that circle. The rotating circle **g** is a semi-meridian, **generatrix**, and the planes perpendicular to axis cut the torus along the concentric circles, called parallels, **d**, which are the **directrices** (guidelines, fig. 148).

Main division of toruses:

1. If the semi-meridian **g** cuts the axis of the surface **a**, torus is **spindled** (fig. 148/a),
2. If the semi-meridian **g** touches the axis of the surface **a**, torus is **thorny** (fig. 148/b),
3. If the semi-meridian **g** does not cut the axis of the surface **a**, torus is **ringed** (fig. 148/c).

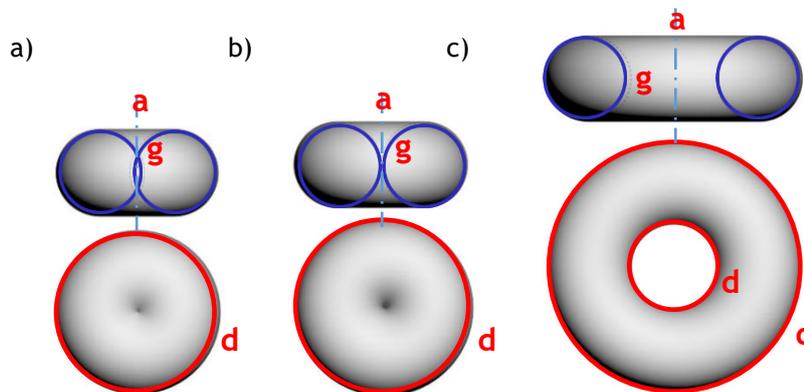


Figure 148 – Division of toruses, a) Spindled; b) Thorny; c) Ringed

## TRANSLATIONAL SURFACES

In translational surfaces both **directrix** (guideline) and **generatrix** are planar curves. Curve **directrix** **d** and curve **generatrix** **g** are given, with one common point **O** (fig. 149). The point **O** moving over the curve **d** goes into the point **L** of that curve by translation movement (parallel transference). By that translation movement the curve **g** is transformed into a curve which is translationally equal to the curve **g**, i.e. it is identical to it. If for all points **L**, of the curve **g**, appropriate translation movement to the curve **d** is applied, then those profile curves **g** cover the translational surface. If both **d** and **g** are planar curves, they cannot belong to the same plane. The same surface is formed by the translation movement of the curve **d** along the curve **g**. On each translational surface there are two systems of generatrix curve of surfaces. Translational surfaces are especially convenient for covering rectangular bases.

Main division of translational surfaces:

1. **general** (fig. 149)
2. **special** - elliptic paraboloids, hyperbolic paraboloids (fig. 150)

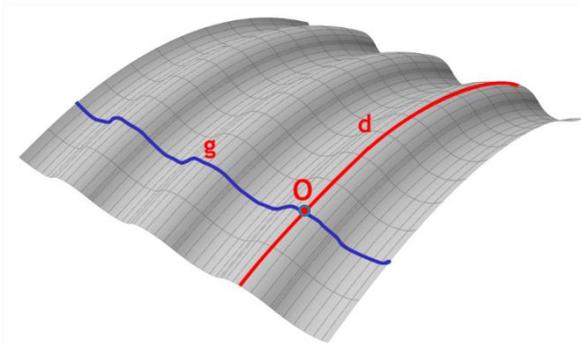


Figure 149 – General translational surface

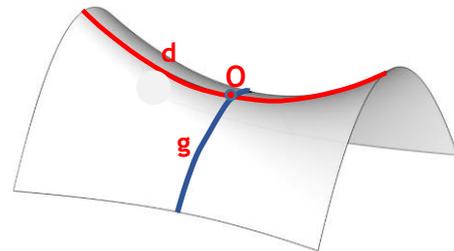


Figure 150 – Special translational surface, HP

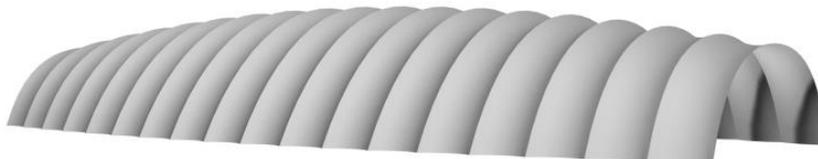


Figure 151 – Example of torus's object, Tarzan Ice Hall, Los Angeles, USA, arch. C. Matson and R. Bradshaw, 1960

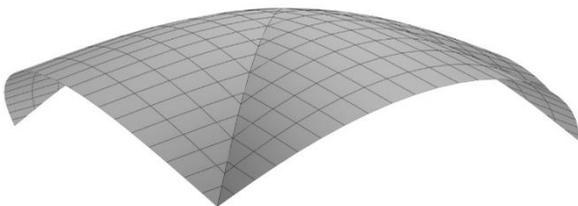


Figure 152 – Example of torus's object, Windward City Mall, Kenneha, USA, arch. R. Bradshaw, 1957

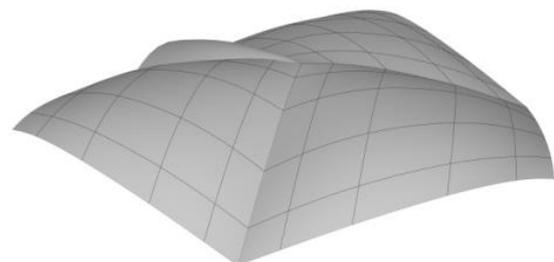


Figure 153 – Example of translational surface object, Airport Reception Building, Las Vegas, USA, arch. W. Becket and R. Bredshaw, 1960

## MODELLING TORUS'S OBJECT AND TRANSLATIONAL SURFACE IN THE COMPUTER SOFTWARE RHINOCEROS

### EXERCISE 12

EXERCISE 12 shows how to construct an object from a ringed torus (fig. 154), in 15 steps, with given dimensions. That is why it is necessary to change measurement units, from mm to m, using the option **File/Properties/Units**. The object consists of the upper half of a ringed torus, which is cut with one regular, four-sided prism, whose center of the square basis coincides with the center of the torus. Firstly, a square was drawn, side 15m, using the option **Curve/Rectangle/Corner to corner** in **Top** view. Torus is modelled using the option **Solid/Torus, Radius 7m, Second Radius 6.9m**, in the same view, and then the bottom half is rejected, best in **Front** view, and the horizontal cutting plane is deleted. Then the torus shell is divided into 12 equal parts using the option **Split**, by vertical cutting planes through the center using the options **Surface/Cutting Plane** and **Transform/Array/ Polar - Number of Items 7, Angle 180<sup>0</sup>**, whereby the center of radial copying is the center of the torus shell (fig. 154/1). In order to make the manipulation easier, all vertical planes and biggest torus shells are hidden, as shown in the figure 154/2. Using the option **Surface/Extrude Curve/Straight** in **Perspective** view, the third dimension was given to the square (four-sided prism was modelled), and then the outside parts of the torus were removed (fig. 154/3). Using the option **Move**, a different height was given to the visible parts of torus shells; those that are above the angles of the square were lowered for 1m, and the other shells were lowered for 2m (fig. 154/4 and 154/5). Then the hidden surfaces were put back (fig. 154/5). Using the option **Mini Menu/Trim**, all inner parts of vertical planes were removed, and all parts of the wall closing the object were preserved (fig. 154/4). The thickness of the shell was determined using the option **Solid /Offset**.

### HOMEWORK 12

HOMEWORK 12 is to model a translational surface (fig. 156), in 3 steps, with given dimensions. That is why it is necessary to change measurement units, from mm to m, using the option **File/Properties/Units**. Using the option **Curve/Free-Form/Interpolate points** two arbitrary curves were drawn, according to figure 156, in **Front**, **Top** and **Perspective** view, having one common point. The curves lie in perpendicular planes. Using the option **Surface/Extrude Curve/Along Curve**, a translational surface was modelled, whereby one curve is given to be a directrix **d**, and the other one is the generatrix **g**. The same surface is generated even if they exchange places (fig. 156).



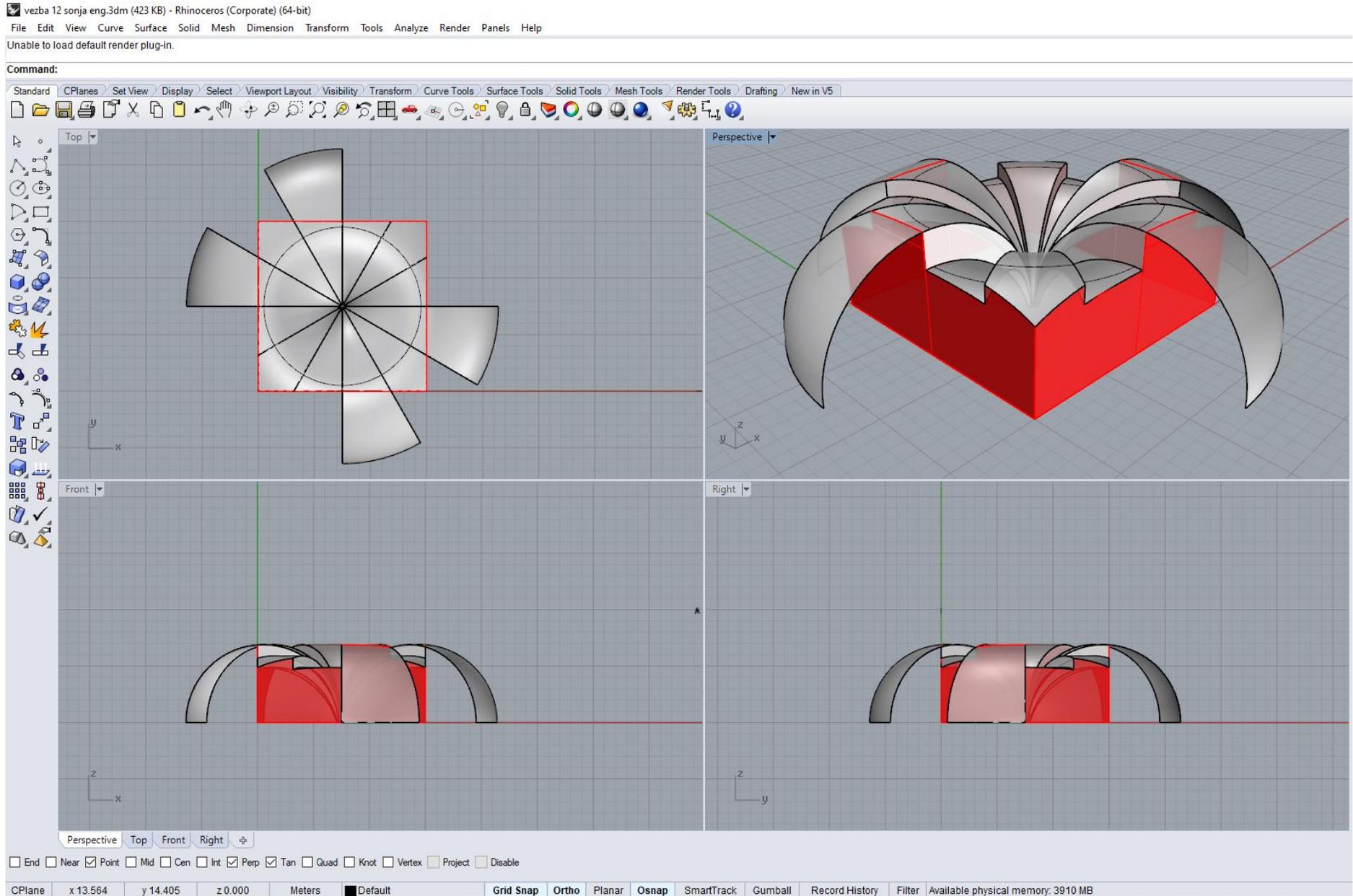


Figure 155 – Example Exercise 12



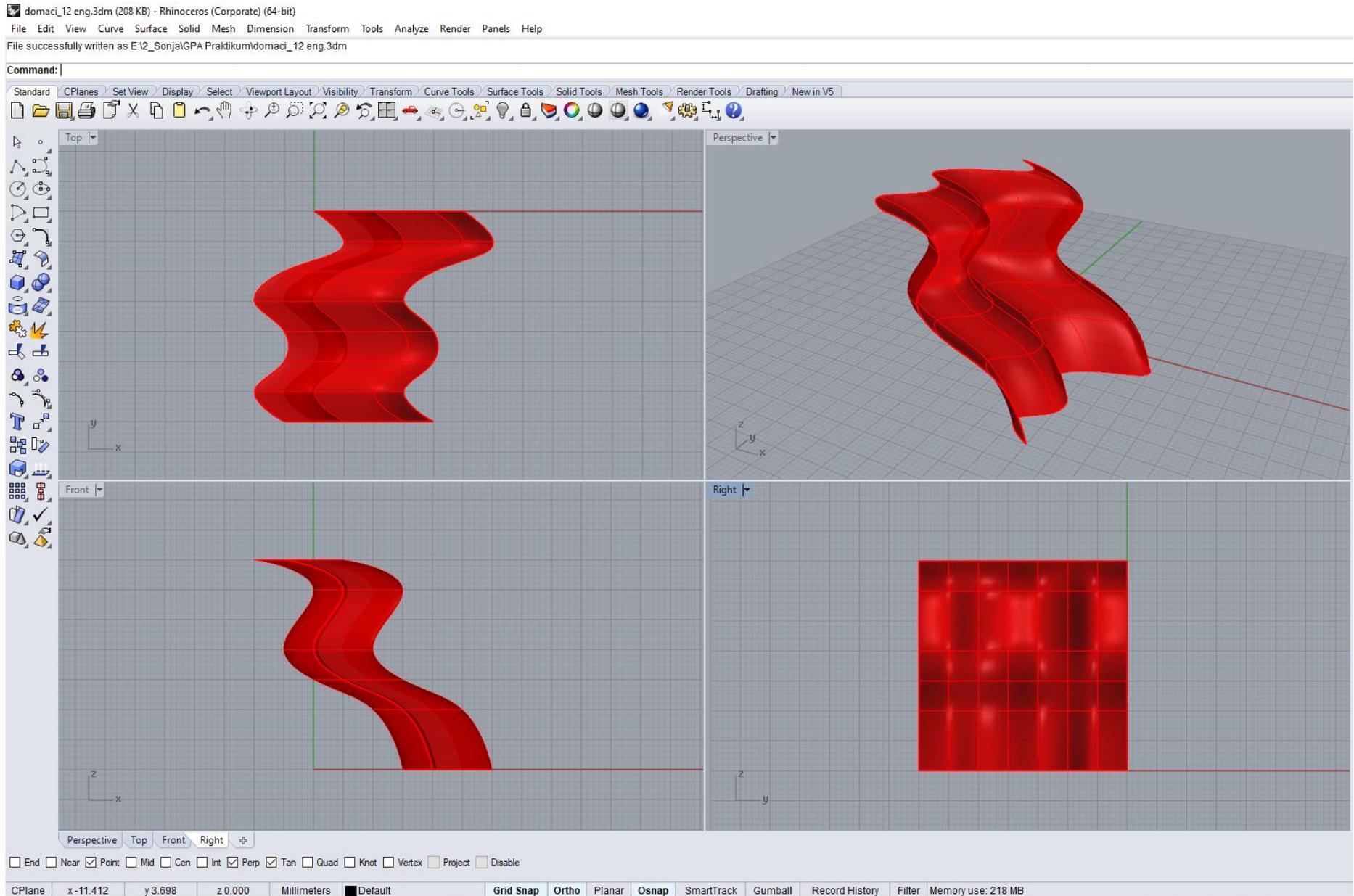


Figure 157 – Example Homework 12

Figures 158 and 159 show representative examples of the use of torus and translational surface in architecture.



Figure 158 – Torus, Aviation Museum, Belgrade, Serbia, arch. Ivan Strauss, year 1965



Figure 159 – Translational surfaces, Airport Reception Building, Las Vegas, USA, arch. W. Becket and R. Bredshaw, year 1960

## **STUDENTS' FINAL PROJECTS**

## Application of pentagonal dodecahedron in architecture

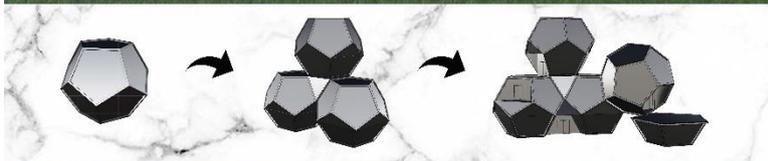


Natalija Savić, Final project, student generation 2016/17

# Application of pentagonal dodecahedron in architecture



PENTAGONALNI DODEKAEDAR



SEVERNA FASADA



ZAPADNA FASADA



JUŽNA FASADA



ISTOČNA FASADA



GRAĐEVINSKO-ARHITEKTONSKI FAKULTET U NIŠU  
GEOMETRIJSKE POVRŠI U ARHITEKTURI

Profesor - dr. Snježana Krsić  
Asistent - Jovana Stanković, Nada Kocić

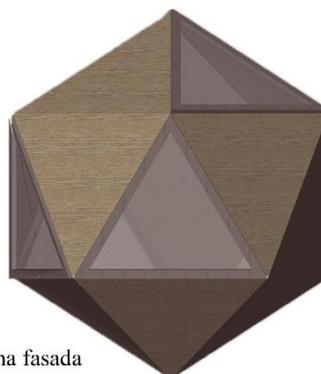
STUDENT - VEDRANA AVRAMOVIĆ RA - 2612

Vedrana Avramović, Final project, student generation 2019/20

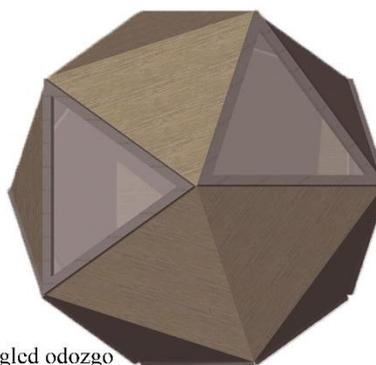
# Application of icosahedron in architecture



## PRIMENA IKOSAEDRA U ARHITEKTURI



Istočna fasada



Pogled odozgo



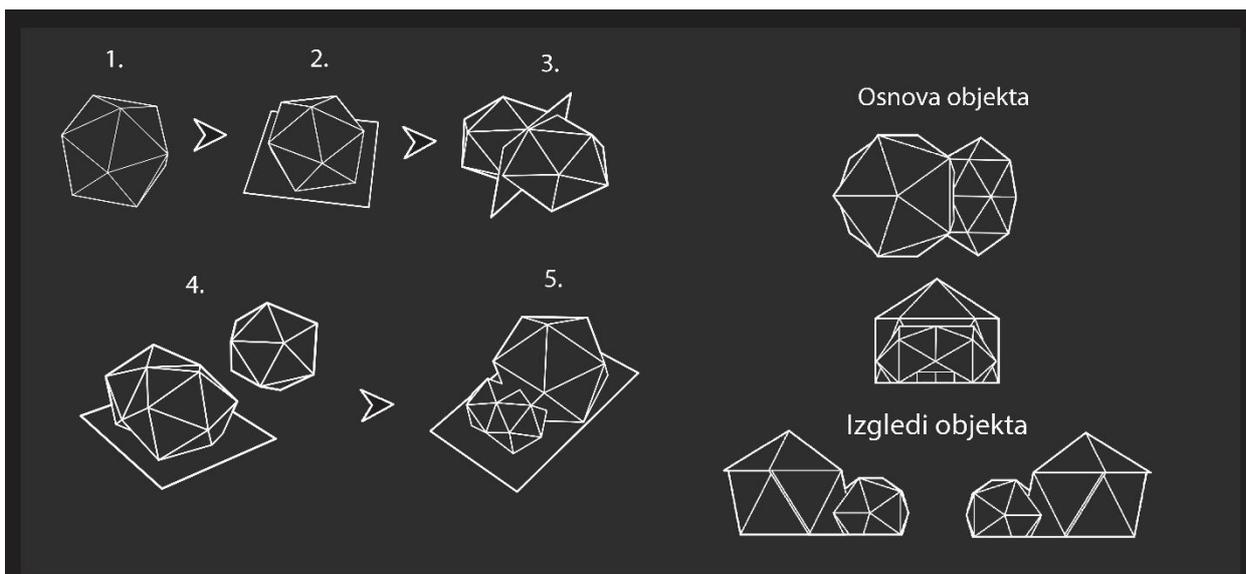
GRADEVINSKO-ARHITEKTONSKI FAKULTET U NIŠU  
GEOMETRIJSKE POVRŠI U ARHITEKTURI  
Profesor: Dr. Sonja Krsić  
Asistent: Nastasija Kocić  
Jovana Krivačević 2775



POSTUPAK DOBIJANJA OBJEKTA

Jovana Krivačević, Final project, student generation 2018/19

## Application of icosahedron in architecture



## Projekat operске zgrade



Građevinsko-arhitektonski fakultet u Nišu  
Geometrijske Površine u Arhitekturi

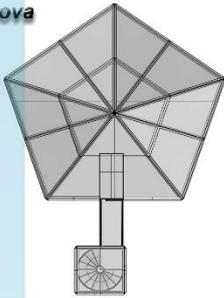
Profesor - dr. Sonja Krsić  
Asistent - Zlata Iosić  
Student - Danijel Golubović 2434



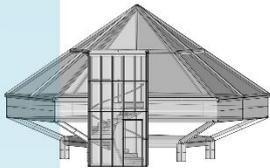
Danijel Golubović, Final project, student generation 2017/18

# Application of pyramids and prisms in architecture

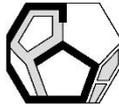
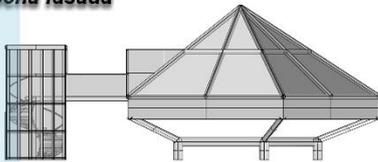
Osnova



Juzna fasada

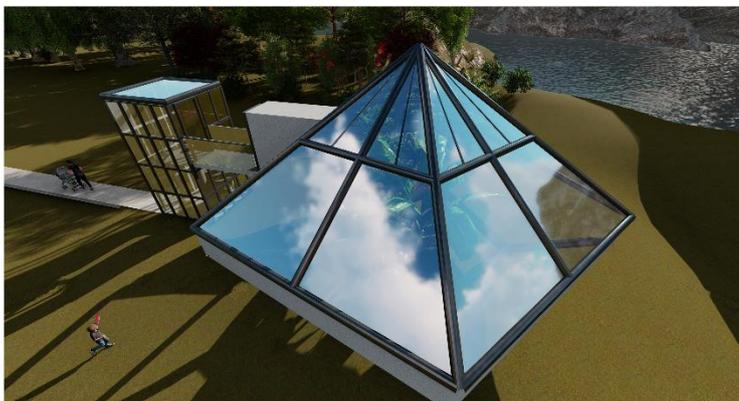
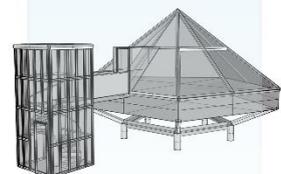
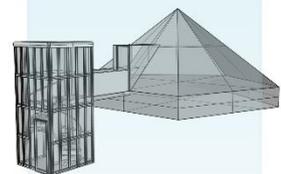
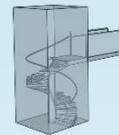


Istocna fasada



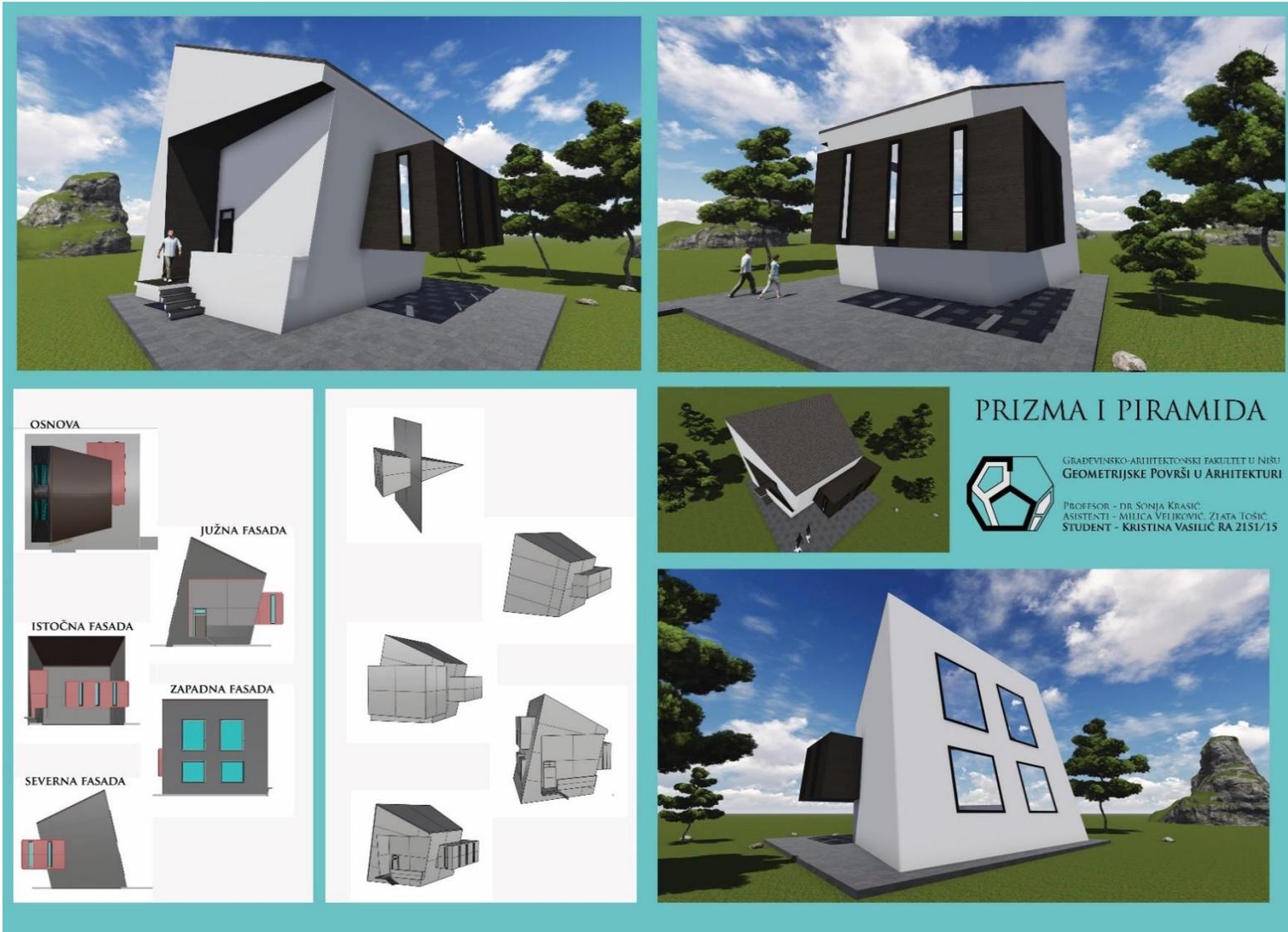
GRAĐEVINSKO-ARHITEKTONSKI FAKULTET U NIŠU  
 GEOMETRIJSKE POVRŠI U ARHITEKTURI  
 PROFESOR - DR SONJA KRASIĆ  
 ASISTENT - ZLATA TOŠIĆ  
 Student - Nevena Kamenović 2507

Postupak dobijanja



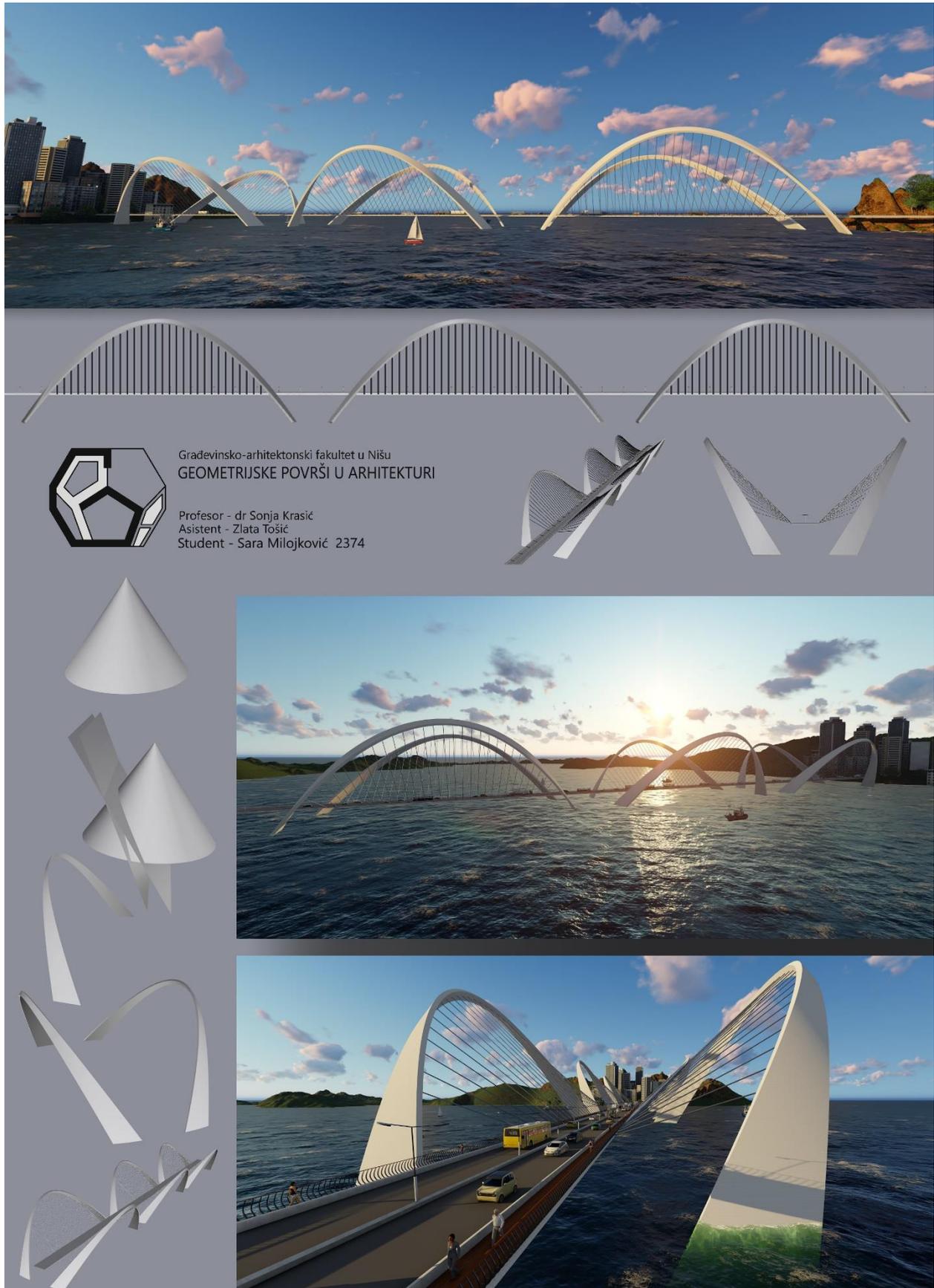
Nevena Kamenović, Final project, student generation 2017/18

## Application of pyramids and prisms in architecture



Kristina Vasić, Final project, student generation 2016/17

## Application of cones and cylinders in architecture



Sara Milojković, Final project, student generation 2017/18

# Application of cones and cylinders in architecture

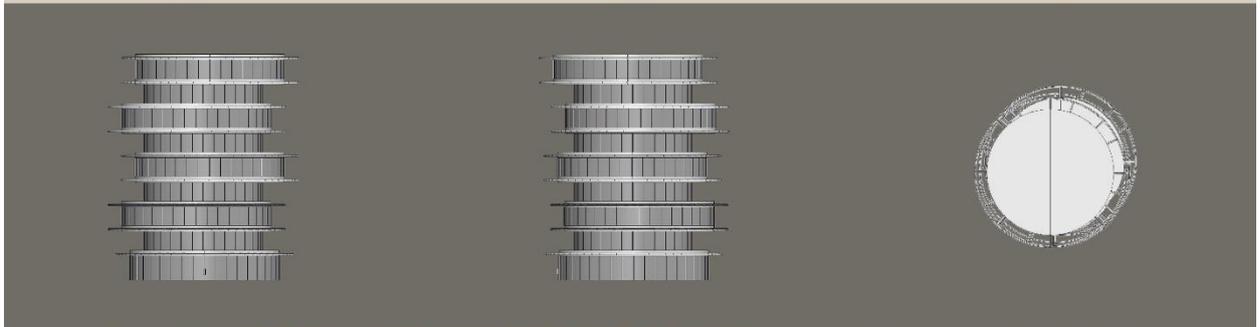
## OBLICA

Javni objekat koji ima funkciju poslovnog centra. Poslovni centar je zamišljen kao "open hub" sa velikim zajedničkim prostorijama na svakoj etaži i pratećim sadržajima kao što su kafe bar, teretana, biblioteka, zona za odmor... Kružna osnova svake etaže i stakleni portal duž cele fasade daju osećaj prostranosti unutar objekta i povezuju enterijer sa okolinom objekta. Objekat je slobodnostojeći, postavljen centralno na platou tako da dominira okolinom i podseća na svojevrstni tip umetničke instalacije. Jezgro objekta i najveći deo noseće konstrukcije su smešteni centralno, gde se nalaze i vertikalne komunikacije, a svaka etaža je konzolno ispuštena različito u odnosu na ostale. Konzolno ispuštene etaže daju razigranost fasadi objekta i stvaraju vid optičke iluzije da svaka od njih stoji nakrivo.



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GEOMETRIJSKE POVRŠI U ARHITEKTURI

Profesor - dr Snežna Krnaić  
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Student - Nina Bojić 2618



3D PRIKAZ

Nina Bojić, Final project, student generation 2019/20

# Application of helicoids in architecture



Marina Jovanović, Final project, student generation 2019/20

# Application of helicoids in architecture



Aleksandar Đorđević, Final project, student generation 2016/17

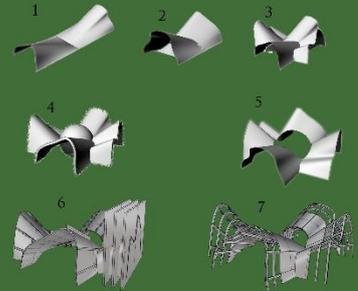
# Application of conoids in architecture



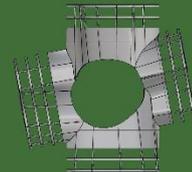
GRAĐEVINSKO ARHITEKTONSKI FAKULTET U NIŠU  
GEOMETRIJSKE POVRŠI U ARHITEKTURI  
PROFESOR - DR. SONJA KRASIĆ  
ASISTENTI - ZLATA TOŠIĆ  
STUDENT - UROŠ ANTIĆ RA-2312

## PRIMENA KONOIDA U ARHITEKTURI

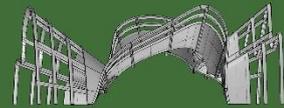
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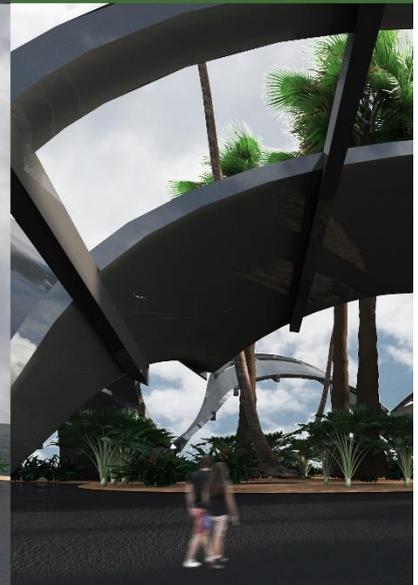
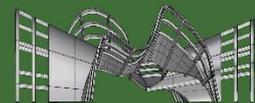
### OSNOVA OBJEKTA



### BOČNI IZGLED



### FRONTALNI IZGLED



Uroš Antić, Final project, student generation 2017/18

# Application of conoids in architecture



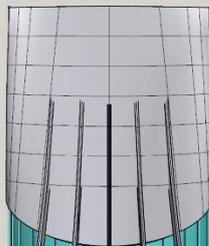
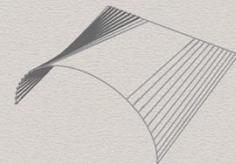
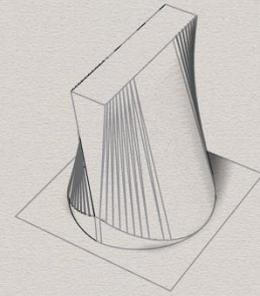
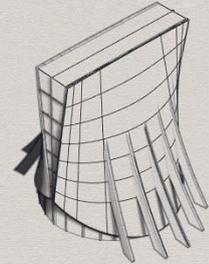
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PROFESOR - DR. SONJA KRASIĆ  
ASISTENT - NASTASIJA KOCIĆ  
STUDENT - NIKOLA KRSTIĆ RA - 2508

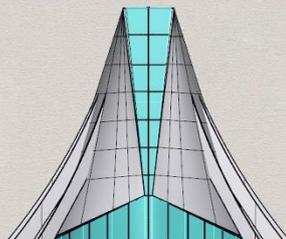
# UPOTREBA KONOIDA U ARHITEKTURI



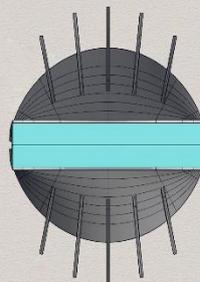
POSTUPAK DOBIJANJA  
FORME ZGRADE



JUŽNA  
FAŠADA



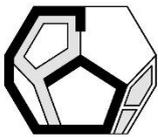
ZAPADNA  
FAŠADA



POGLED  
ODOZGO

Nikola Krstić, Final project, student generation 2018/19

# Application of cylindroids in architecture

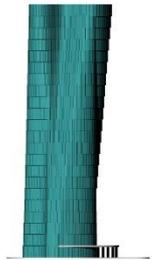


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Profesor - dr. Sonja Krsić  
Asistent - Nastasija Kocić  
Student - Predrag Mihajlović RA-2679

IZGLEDI

OSNOVA



POSTUPAK



Predrag Mihajlović, Final project, student generation 2019/20

## Application of cylindroids in architecture



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GEOMETRIJSKE POVRŠI U ARHITEKTURI

Profesor - dr. Sonja Krsić  
Asistent - Jovana Stanković, Nada Kocić  
Student - Kristina Pešić RA-2769

# PRIMENA CILINDROIDA U ARHITEKTURI

### KARAKTERISTIČNI KORACI DOBIJENOG OBJEKTA



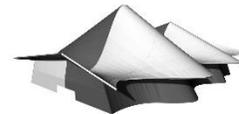
KORAK 1



KORAK 2



KORAK 3



KORAK 4



KORAK 5

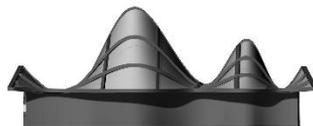


3D PRIKAZ OBJEKTA

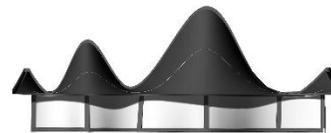
### KARAKTERISTIČNI ORTOGONALNI IZGLEDI



SEVERNA FASADA



ISTOČNA FASADA



ZAPADNA FASADA



JUŽNA FASADA

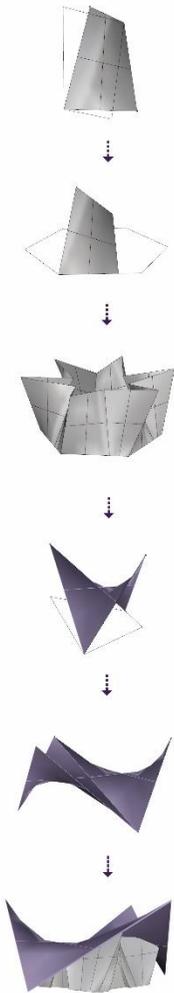


POGLED ODOZGO

Kristina Pešić, Final project, student generation 2019/20

# Application of hyperbolic paraboloids in architecture

KORACI  
U DOBIJANJU FORME OBJEKTA



ZAPADNA FASADA R 1:500



SEVERNA FASADA R 1:500



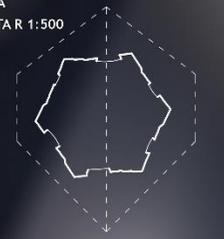
ISTOČNA FASADA R 1:500



JUŽNA FASADA R 1:500



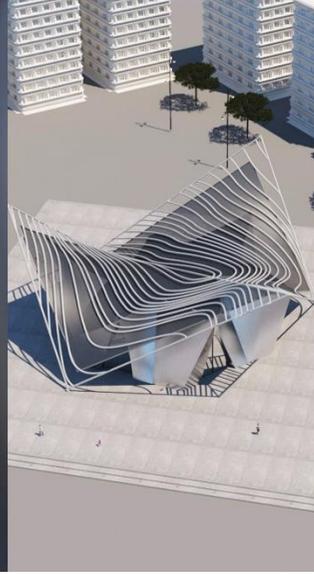
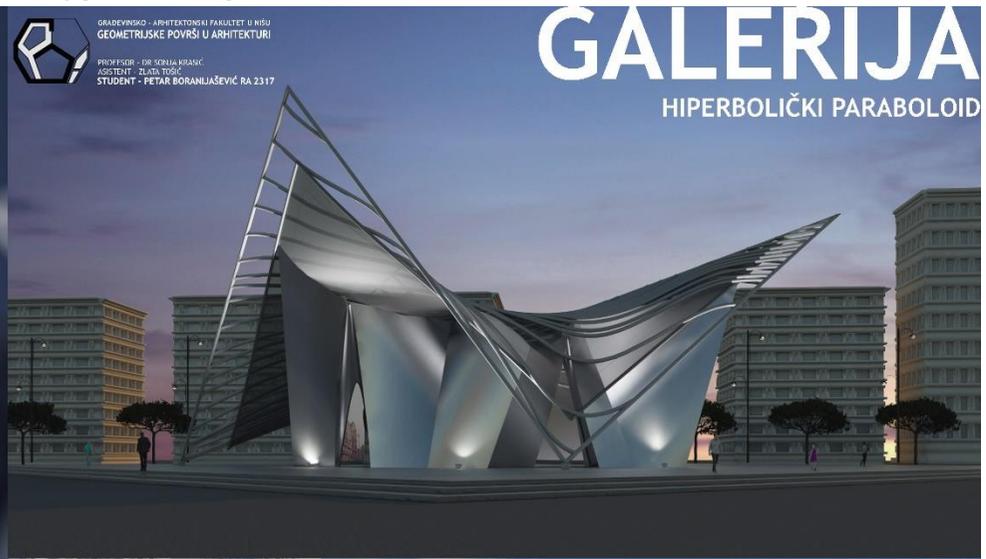
OSNOVA  
OBJEKTA R 1:500



GRAĐEVINSKO - ARHITEKTONSKI FAKULTET U RIŠU  
GEOMETRIJSKE POVRŠI U ARHITEKTURI  
PROFESOR - DR. SERŽIJA HRŠIĆ  
ASISTENT - BLAŽA TOČIĆ  
STUDENT - PETAR BORANIJAŠEVIĆ RA 2317

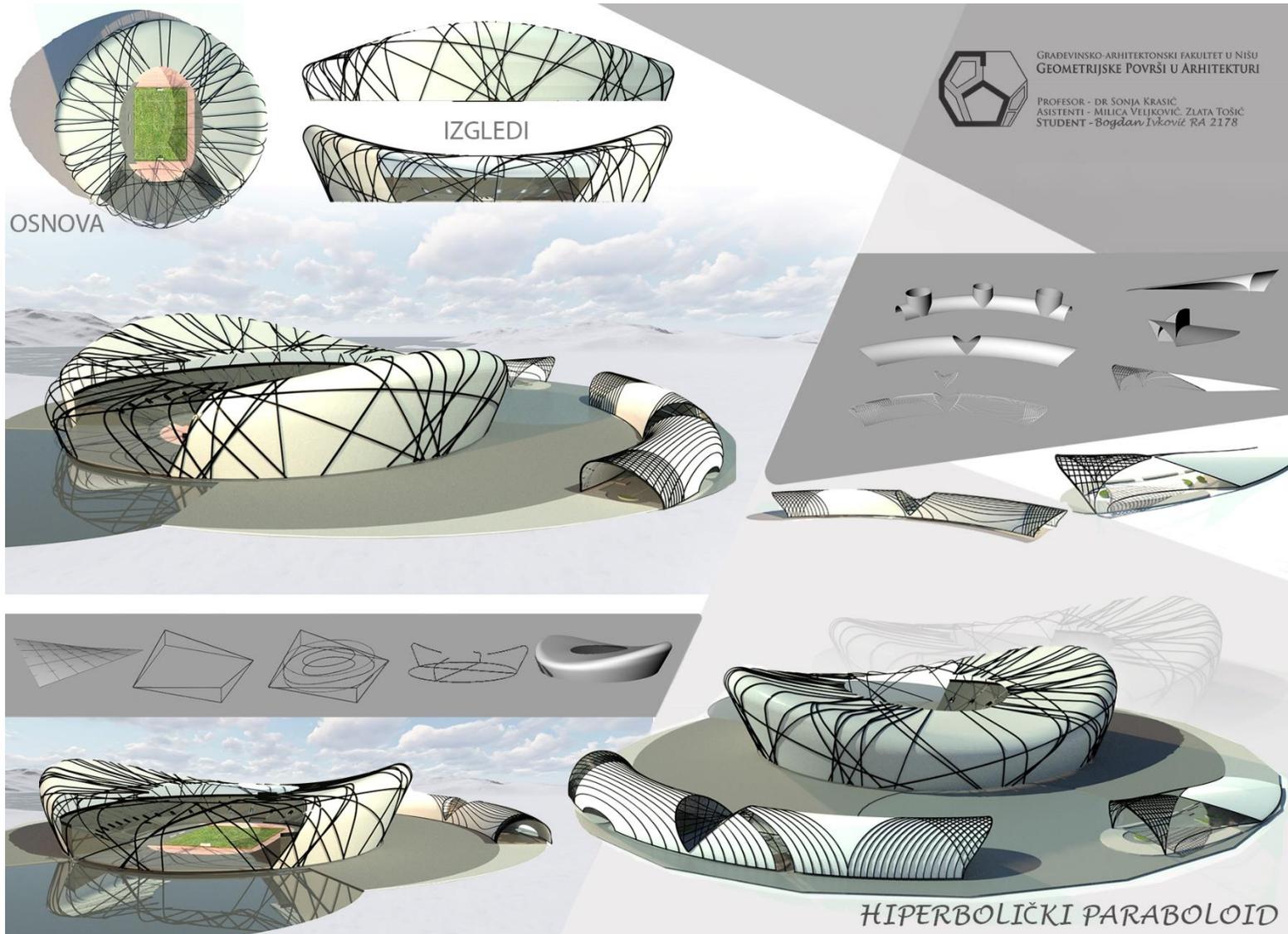
# GALERIJA

HIPERBOLIČKI PARABOLOID



Petar Boranijašević, Final project, student generation 2017/18

# Application of hyperbolic paraboloids in architecture



GRAĐEVINSKO-ARHITEKTONSKI FAKULTET U NIŠU  
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PROFESOR - DR. SONJA KRASIĆ  
ASISTENTI - MILICA VELJKOVIĆ, ZLATA TOŠIĆ  
STUDENT - *Bogdan Ivković* RA 2178

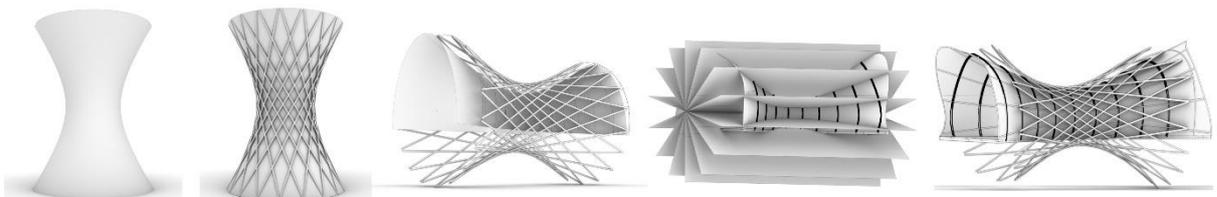
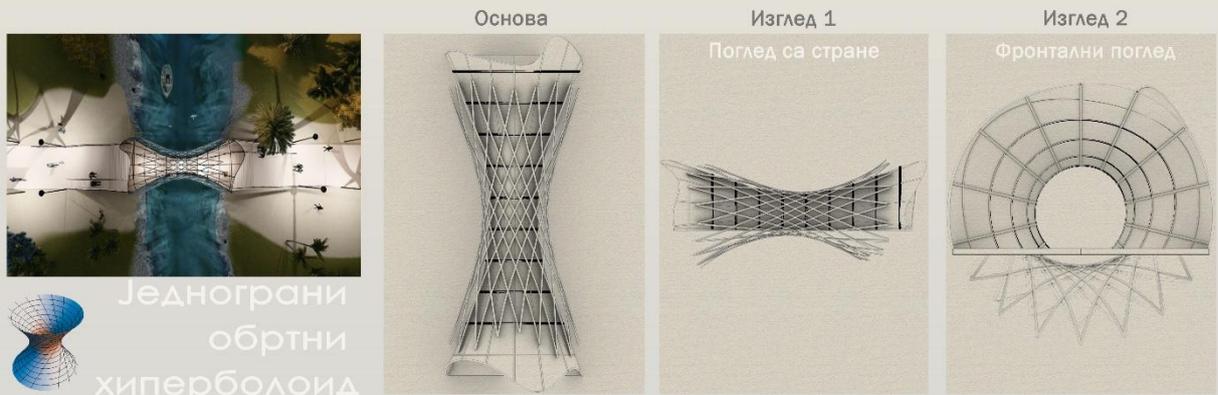
Bogdan Ivković, Final project, student generation 2016/17

# Application of one-sheet rotational hyperboloid in architecture



GRAĐEVINSKO-ARHITEKTONSKI FAKULTET U NIŠU  
GEOMETRIJSKE POVRŠI U ARHITEKTURI

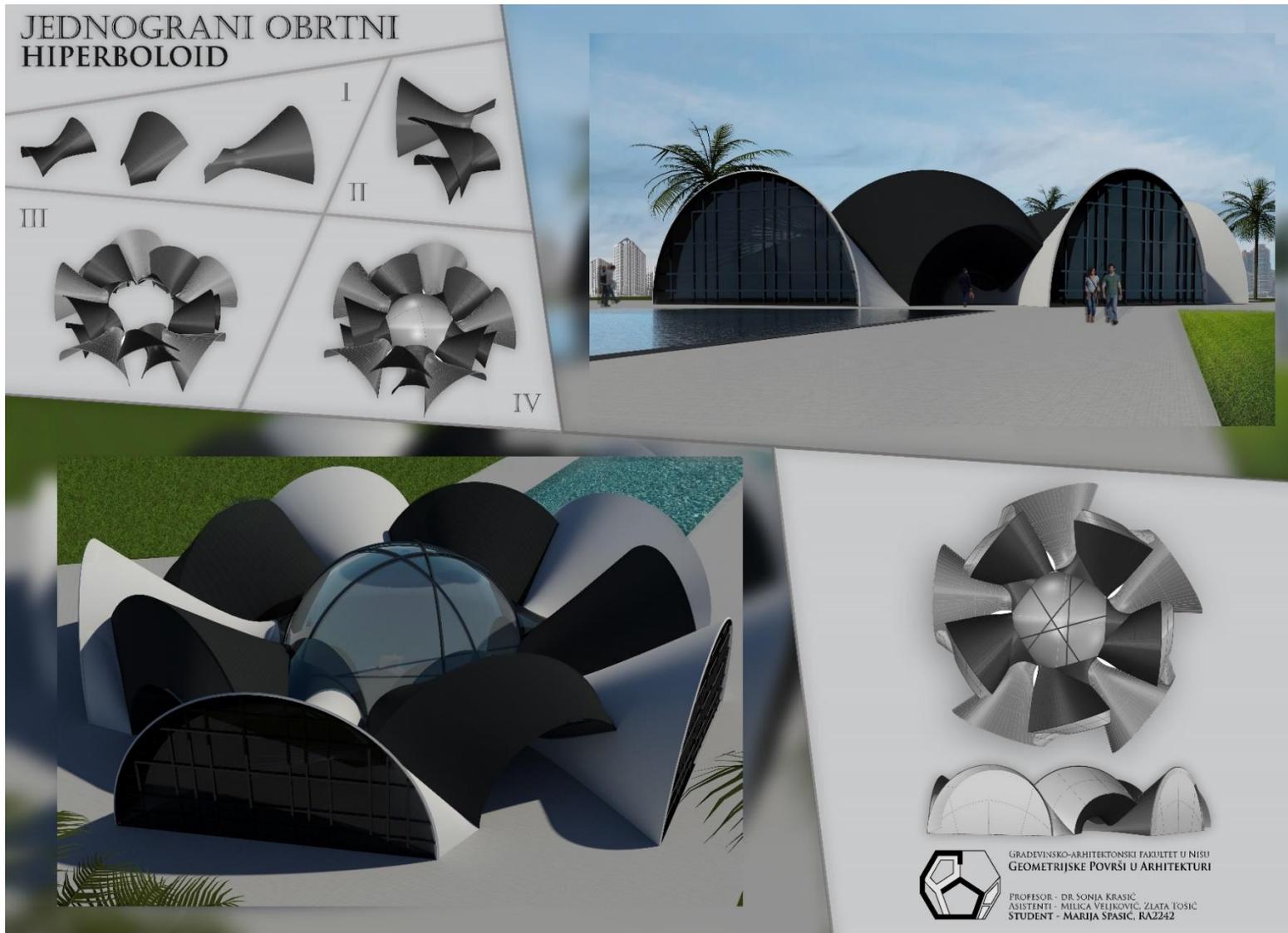
PROFESOR - DR. SONJA KRSTIĆ  
ASISTENT - NADIJA KOŠIĆ  
JANA KOSTIĆ 2656



Корази у моделовању објекта

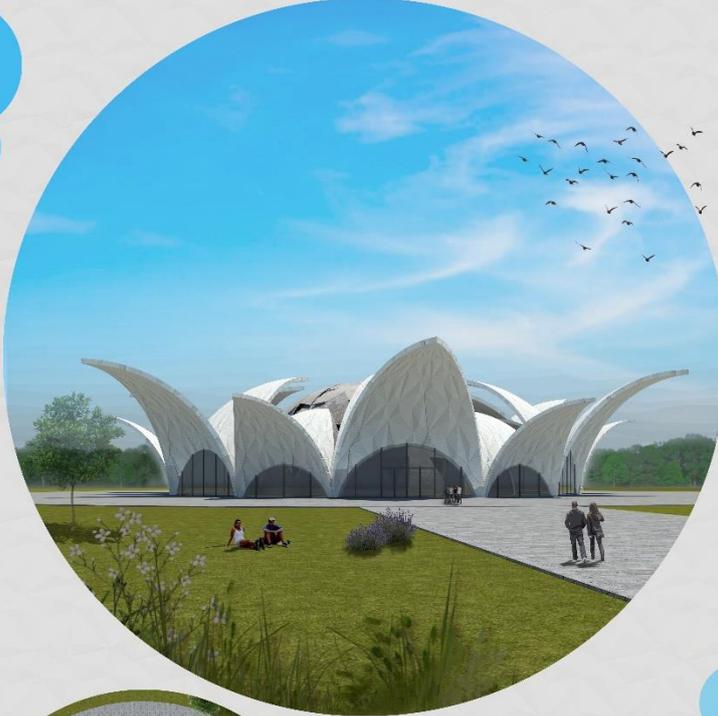
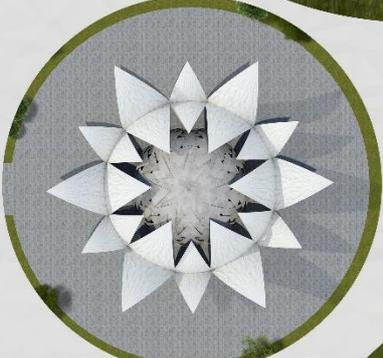
Jana Kostić, Final project, student generation 2019/20

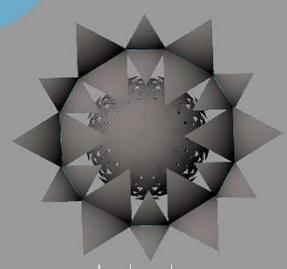
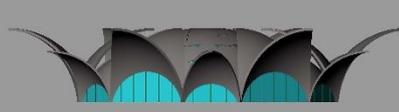
## Application of one-sheet rotational hyperboloid in architecture



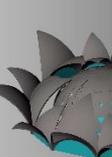
Marija Spasić, Final project, student generation 2016/17

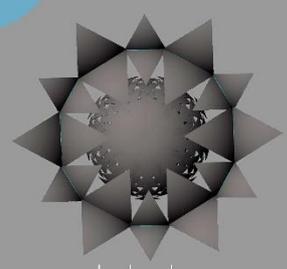
# Application of sphere in architecture

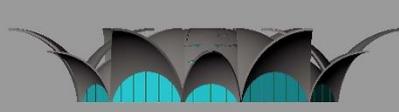



postupak

1. korak 
2. korak 
3. korak 
4. korak 
5. korak  
6. korak 
7. korak 
8. korak 
9. korak 
10. korak 



pogled odozgo



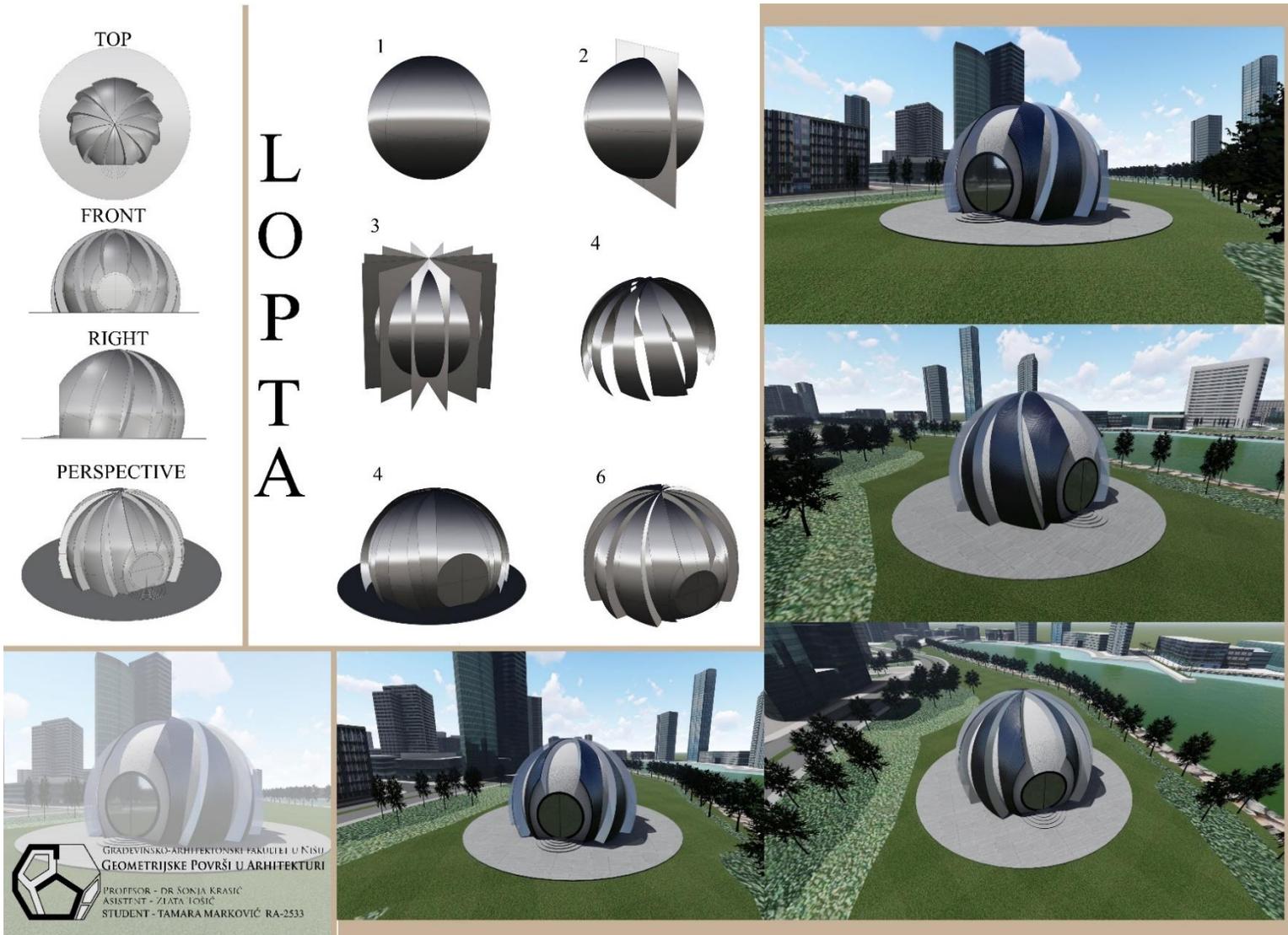
pogled sa strane

## SØERA

GRAĐEVINSKO-ARHITEKTONSKI FAKULTET U NISU  
 GEOMETRIJSKE POVRŠI U ARHITEKTURI  
 PROFESOR – DR. SONJA KRASIĆ  
 ASISTENT – ZLATA TOŠIĆ  
 Student – Marijana Minić RA2525

Marijana Minić, Final project, student generation 2018/19

## Application of sphere in architecture



Tamara Marković, Final project, student generation 2018/19

# Application of rotational ellipsoid in architecture



GRAĐEVINSKO-ARHITEKTONSKI FAKULTET U NIŠU  
GEOMETRIJSKE POVRŠI U ARHITEKTURI

Profesor - dr Sonja Krsić  
Asistent -  
Student - Nina Bogojević RA-2617

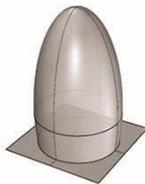
OSNOVA



IZGLED



POSTUPAK



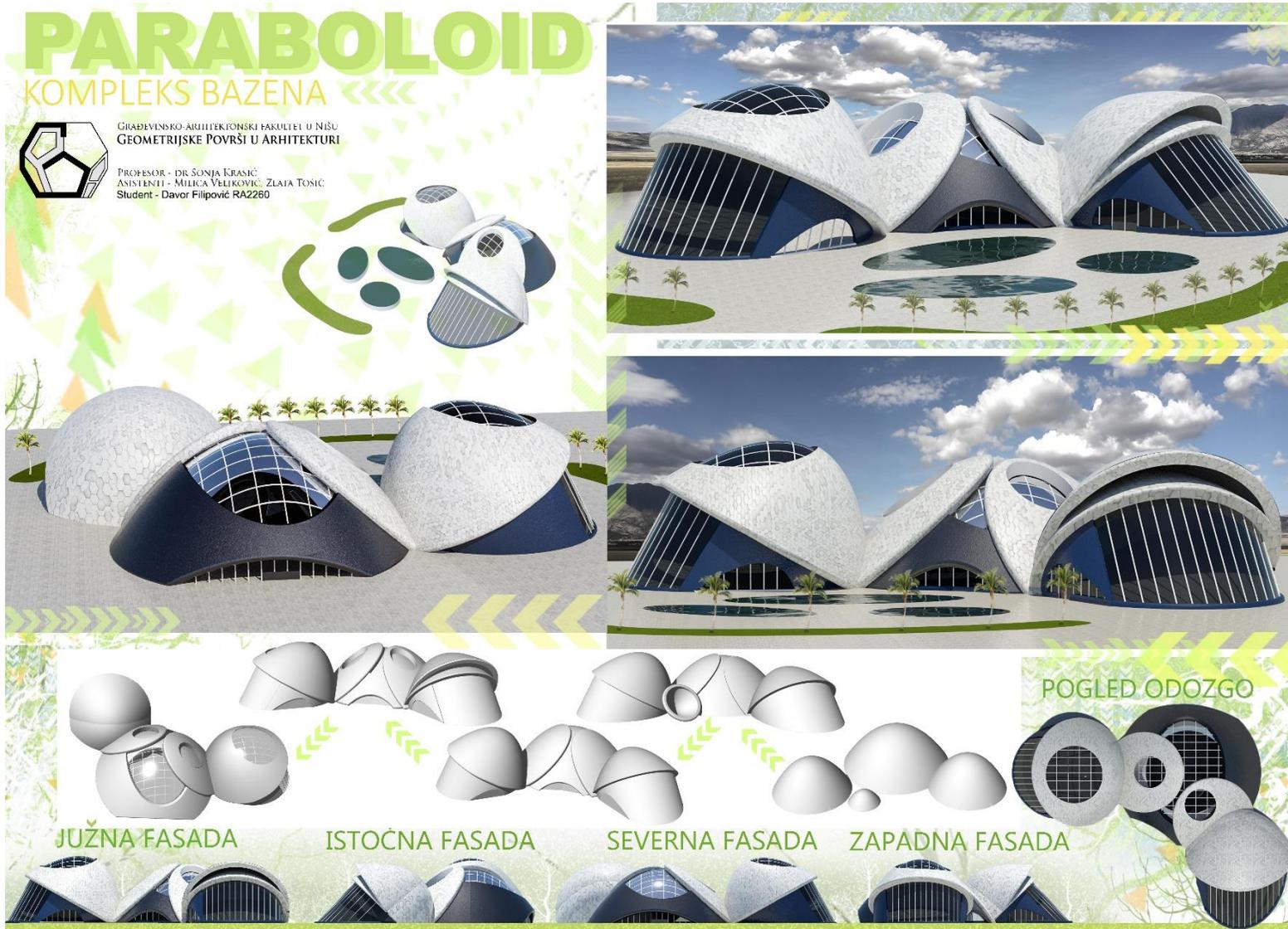
Nina Bogojević, Final project, student generation 2019/20

# Application of triaxial ellipsoid in architecture



Nikola Minić, Final project, student generation 2018/19

# Application of rotational paraboloid in architecture



Davor Filipović, Final project, student generation 2016/17

## Application of rotational paraboloid in architecture



GRADIVINSKO-ARHITEKTONSKI FAKULTET U NIŠU  
GEOMETRIJSKE POVRŠI U ARHITEKTURI

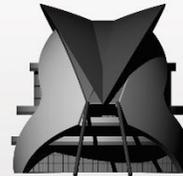
PROFESOR - DR. SONJA KRASIĆ  
ASISTENT - JOVANA STANKOVIĆ  
STUDENT - DUŠAN STAMENKOVIĆ 2609

PARABOLOID

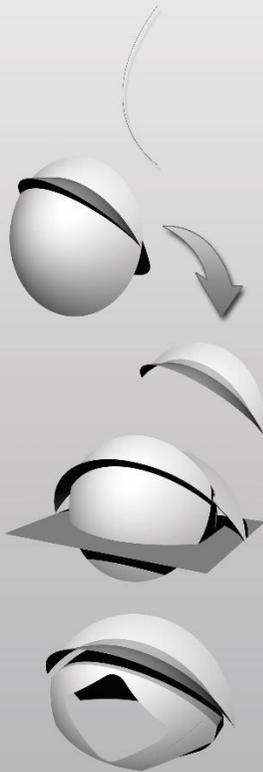
OSNOVA



IZGLED 1



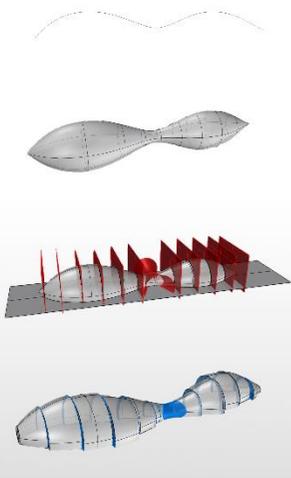
IZGLED 2



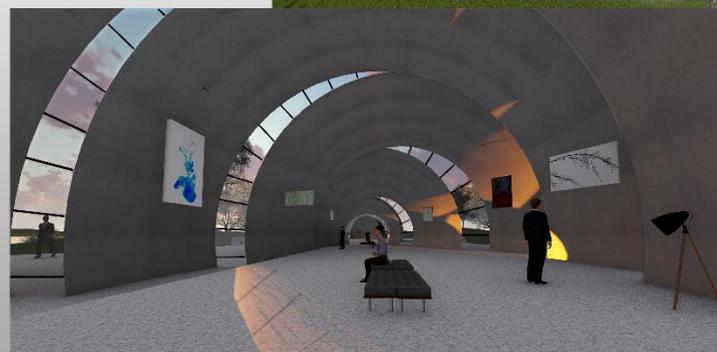
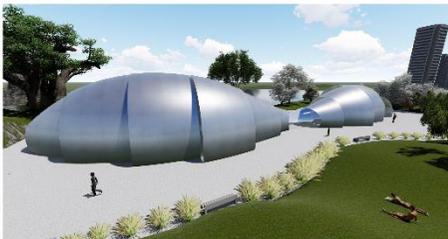
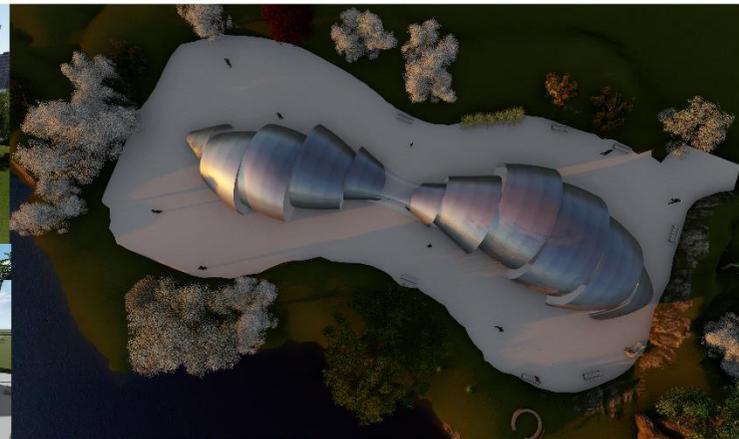
Dušan Stamenković, Final project, student generation 2018/19

# Application of general type of rotational surface in architecture

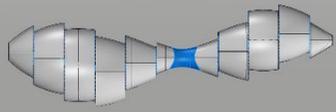
**POSTUPAK**



**OPŠTA OBRтна POVRŠ**



**OSNOVA**



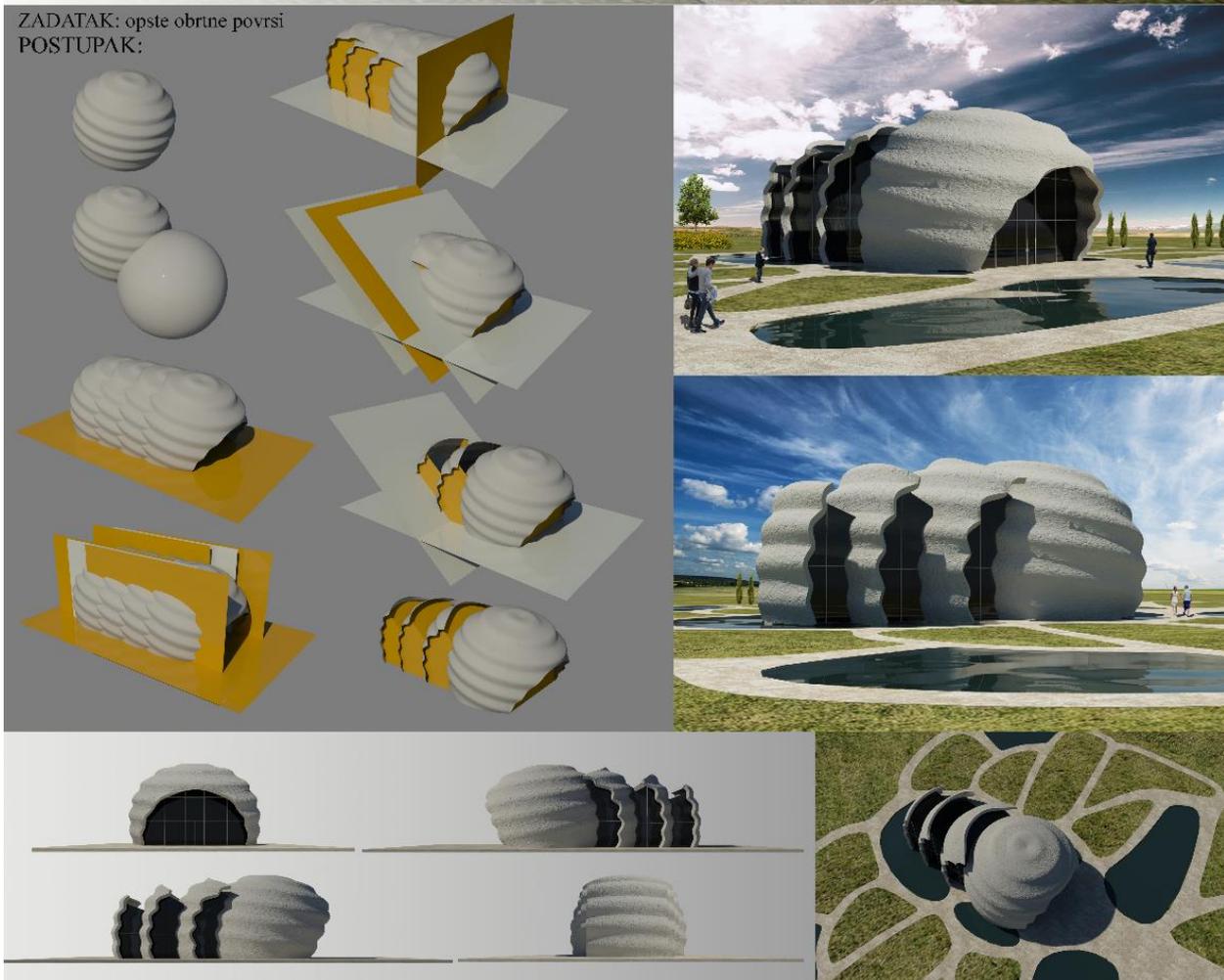
**IZGLEDI**



**GRAĐEVINSKO-ARHITEKTONSKI FAKULTET U NIŠU  
GEOMETRIJSKE POVRŠI U ARHITEKTURI**  
Profesor - dr. Sonja Krsajić  
Asistent - Nastasija Kocić  
Student - Martin Stojančev 2569

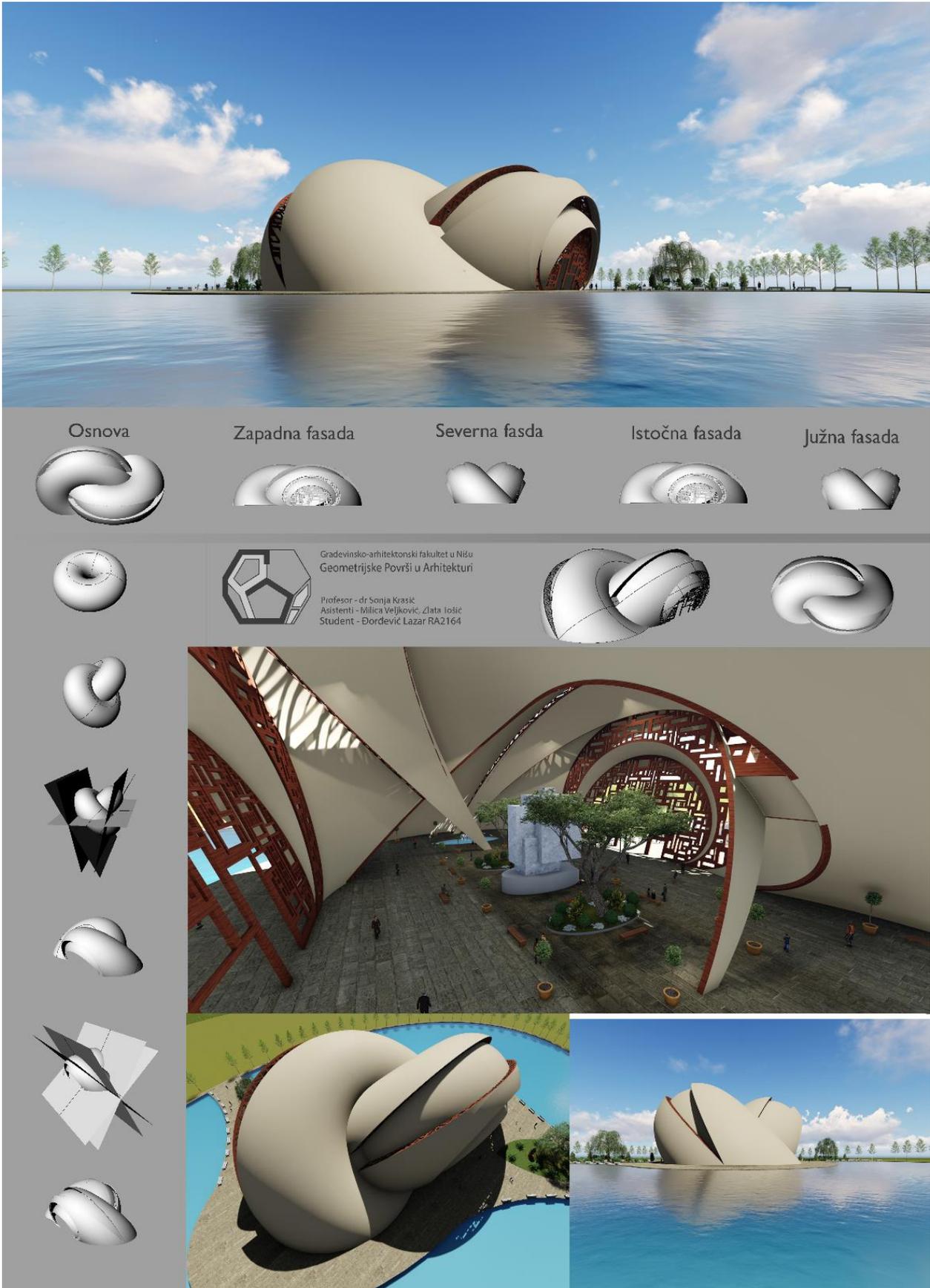
Martin Stojančev, Final project, student generation 2018/19

## Application of general type of rotational surface in architecture



Tijana Stanković, Final project, student generation 2016/17

# Application of torus in architecture



Lazar Đorđević, Final project, student generation 2016/17

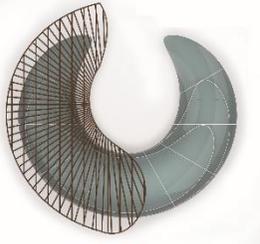
# Application of torus in architecture



Građevinsko arhitektonski fakultet u Nišu  
**Geometrijske Površine u Arhitekturi**  
Profesor - dr. Sanja Krasić  
Asistent - Nostosija Kocić  
Student - Milenković Mihajlo, 2675

## TORUS

OSNOVA OBJEKTA



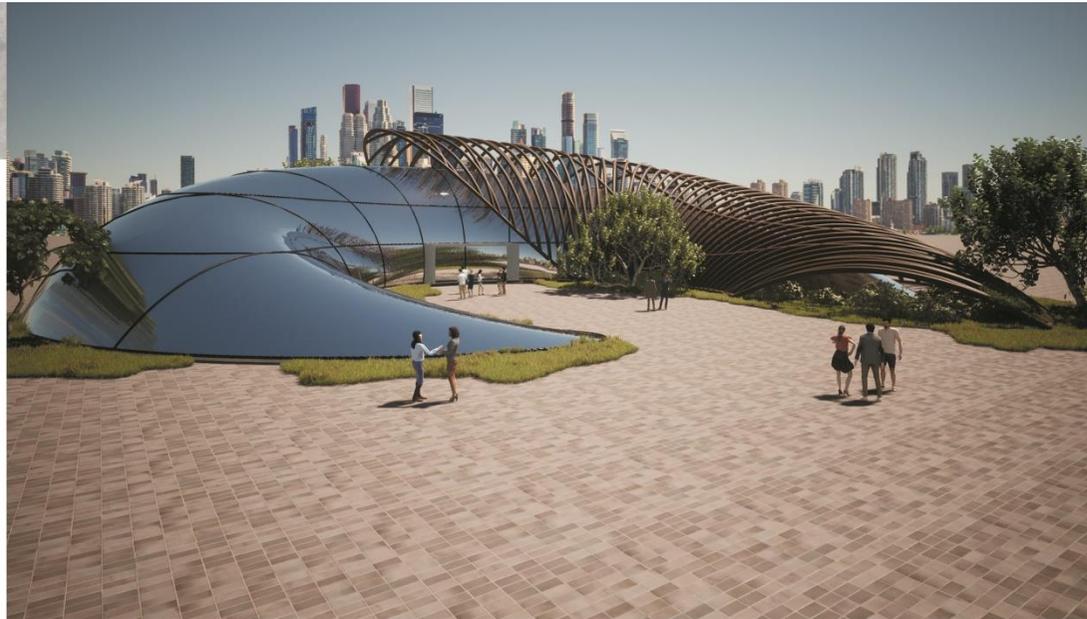
FASADA 1



FASADA 2



POSTUPAK DOBIJANJA FORME

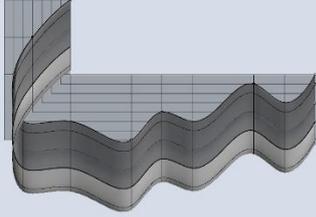


Mihajlo Milenković, Final project, student generation 2019/20

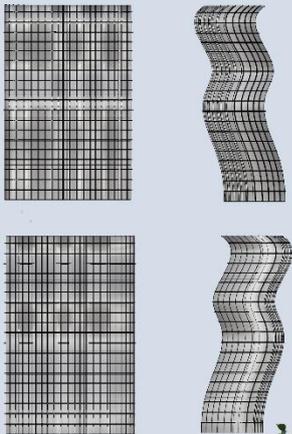
# Application of translational surfaces in architecture

## PRIMENA KLIZNIH POVRŠI U ARHITEKTURI

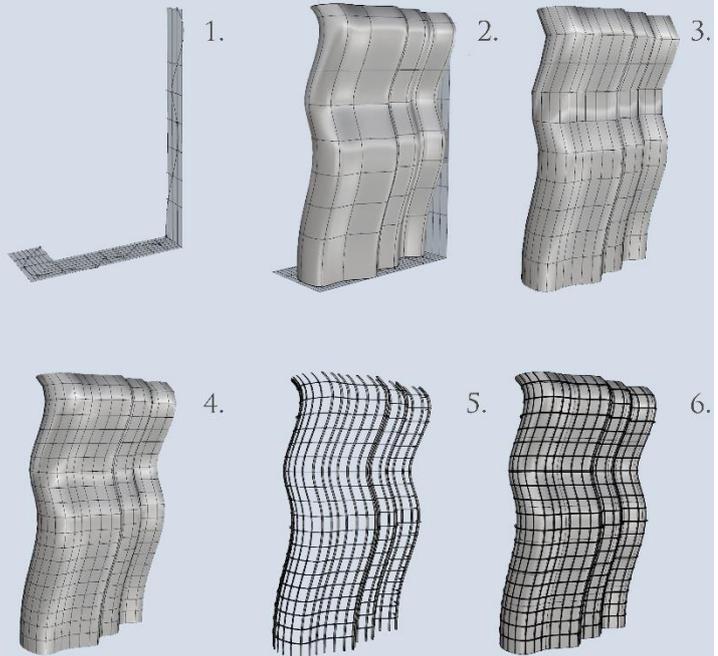
### OSNOVA KLIZNE POVRŠI



### IZGLEDI



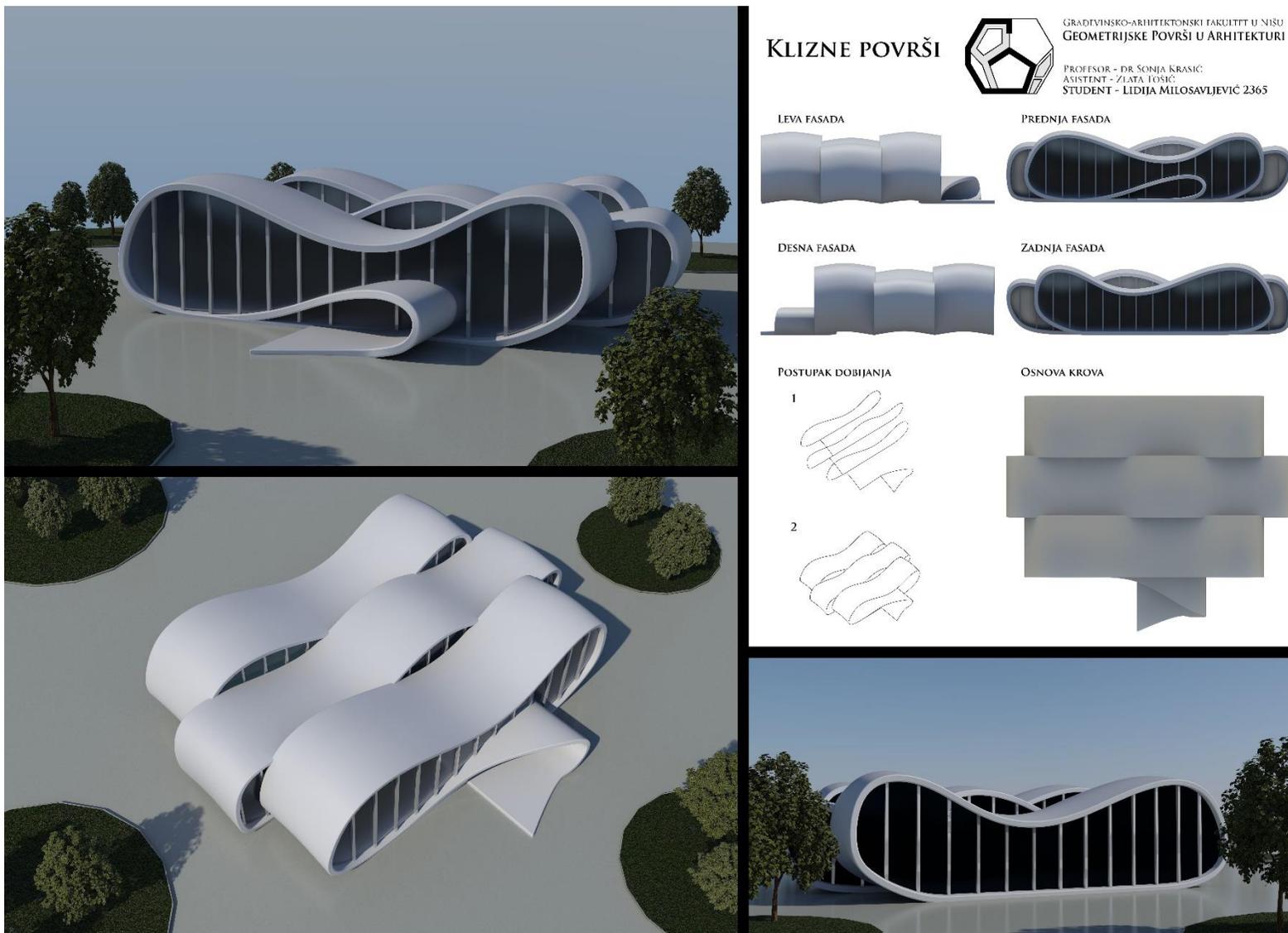
### KORACI ZA DOBIJANJE KLIZNE POVRŠI



GEOMETRIJSKE ARHITEKTONSKE FASADE U A.S.4  
GEOMETRIJSKE POVRŠI U ARHITEKTURI  
PROFESOR - DR. SONJA UPAŠIĆ  
ASISTENT - Z. IVA TEŠIĆ  
STUDENT - STOJILJKOVIĆ NIKOLA 2375

Nikola Stojiljković, Final project, student generation 2018/19

## Application of translational surfaces in architecture



Lidija Milosavljević, Final project, student generation 2017/18

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## Review of a manuscript entitled

### Geometrical surfaces in architecture – practice book

authors PhD Sonja Krasić and MSc Zlata Tošić

"Geometrical surfaces in architecture – practice book" is a textbook for students of architecture to learn about geometrical forms in architecture. Through 12 units, students will learn the characteristics of various forms while creating 3D models.

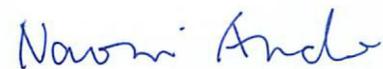
In the first unit, students will learn how to use Rhinoceros, the 3D modeling software used in this book. In the second and third units, they will learn how to create basic forms. In the fourth to sixth units, they will learn the principles of solid primitives such as polyhedrons, and in the seventh and subsequent units, they will learn how to create various geometrical surfaces. In addition, exercises are provided for each unit to allow for thorough review.

A key feature of this book is that it introduces many examples of attractive architecture composed of various forms, encourage students to create such a variety of forms while understanding significant architectural configurations.

In order to create various forms freely by Rhinoceros, it is necessary to get used to its operations. However, in my opinion, what is important in design is not “How to do”, but “What to do”. When students use the software with a creative mind, they will learn how to use it as they go along. This book, which explains the composition of geometrical surfaces in fascinating architecture from around the world, will bring hints for the next architectural design and enrich the ideas of students.

Tokyo, Japan

8/6/2021



Prof. Naomi Ando, Dr. Eng.  
Department of Architecture  
Faculty of Engineering and Design  
Hosei University, Tokyo, Japan

**GEOMETRICAL SURFACES IN ARCHITECTURE – PRACTICE BOOK**

Authors: PhD Соња Красић, MSc Злата Тошић

Review: Prof. Naomi Ando, Dr. Eng.

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